Foundations of Mathematics, Grade 10, Applied (MFM2P)

This course enables students to consolidate their understanding of linear relations and extend their problem-solving and algebraic skills through investigation, the effective use of technology, and hands-on activities. Students will develop and graph equations in analytic geometry; solve and apply linear systems, using real-life examples; and explore and interpret graphs of quadratic relations. Students will investigate similar triangles, the trigonometry of right triangles, and the measurement of three-dimensional figures. Students will consolidate their mathematical skills as they solve problems and communicate their thinking.

Mathematical process expectations. The mathematical processes are to be integrated into student learning in all areas of this course.

Throughout this course, students will:

PROBLEM SOLVING
- develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;

REASONING AND PROVING
- develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counter-examples) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;

REFLECTING
- demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);

SELECTING TOOLS AND COMPUTATIONAL STRATEGIES
- select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;

CONNECTING
- make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);

REPRESENTING
- create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems;

COMMUNICATING
- communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

Measurement and Trigonometry

Overall Expectations

By the end of this course, students will:

- use their knowledge of ratio and proportion to investigate similar triangles and solve problems related to similarity;
- solve problems involving right triangles, using the primary trigonometric ratios and the Pythagorean theorem;
- solve problems involving the surface areas and volumes of three-dimensional figures, and use the imperial and metric systems of measurement.

Specific Expectations

Solving Problems Involving Similar Triangles

By the end of this course, students will:

– verify, through investigation (e.g., using dynamic geometry software, concrete materials), properties of similar triangles (e.g., given similar triangles, verify the equality of corresponding angles and the proportionality of corresponding sides);

– determine the lengths of sides of similar triangles, using proportional reasoning;

– solve problems involving similar triangles in realistic situations (e.g., shadows, reflections, scale models, surveying) (***Sample problem:*** Use a metre stick to determine the height of a tree, by means of the similar triangles formed by the tree, the metre stick, and their shadows.).

Solving Problems Involving the Trigonometry of Right Triangles

By the end of this course, students will:

– determine, through investigation (e.g., using dynamic geometry software, concrete materials), the relationship between the ratio of two sides in a right triangle and the ratio of the two corresponding sides in a similar right triangle, and define the sine, cosine, and tangent ratios (e.g., $\sin A = \frac{opposite}{hypotenuse}$);

– determine the measures of the sides and angles in right triangles, using the primary trigonometric ratios and the Pythagorean theorem;

– solve problems involving the measures of sides and angles in right triangles in real-life applications (e.g., in surveying, in navigation, in determining the height of an inaccessible object around the school), using the primary trigonometric ratios and the Pythagorean theorem (***Sample problem:*** Build a kite, using imperial measurements, create a clinometer to determine the angle of elevation when the kite is flown, and use the tangent ratio to calculate the height attained.);

– describe, through participation in an activity, the application of trigonometry in an occupation (e.g., research and report on how trigonometry is applied in astronomy; attend a career fair that includes a surveyor, and describe how a surveyor applies trigonometry to calculate distances; job shadow a carpenter for a few hours, and describe how a carpenter uses trigonometry).

Solving Problems Involving Surface Area and Volume, Using the Imperial and Metric Systems of Measurement

By the end of this course, students will:

– use the imperial system when solving measurement problems (e.g., problems involving dimensions of lumber, areas of carpets, and volumes of soil or concrete);

– perform everyday conversions between the imperial system and the metric system (e.g., millilitres to cups, centimetres to inches) and within these systems (e.g., cubic metres to cubic centimetres, square feet to square yards), as necessary to solve problems involving measurement (***Sample problem:*** A vertical post is to be supported by a wooden pole, secured on the ground at an angle of elevation of 60°, and reaching 3 m up the post from its base. If wood is sold by the foot, how many feet of wood are needed to make the pole?);

– determine, through investigation, the relationship for calculating the surface area of a pyramid (e.g., use the net of a square-based pyramid to determine that the surface area is the area of the square base plus the areas of the four congruent triangles);

– solve problems involving the surface areas of prisms, pyramids, and cylinders, and the volumes of prisms, pyramids, cylinders, cones, and spheres, including problems involving combinations of these figures, using the metric system or the imperial system, as appropriate (***Sample problem:*** How many cubic yards of concrete are required to pour a concrete pad measuring 10 feet by 10 feet by 1 foot? If poured concrete costs $110 per cubic yard, how much does it cost to pour a concrete driveway requiring 6 pads?).

Modelling Linear Relations

Overall Expectations

By the end of this course, students will:

- manipulate and solve algebraic equations, as needed to solve problems;
- graph a line and write the equation of a line from given information;
- solve systems of two linear equations, and solve related problems that arise from realistic situations.

Specific Expectations

Manipulating and Solving Algebraic Equations

By the end of this course, students will:

– solve first-degree equations involving one variable, including equations with fractional coefficients (e.g. using the balance analogy, computer algebra systems, paper and pencil) (***Sample problem:*** Solve $\frac{x}{2} + 4 = 3x - 1$ and verify.);

– determine the value of a variable in the first degree, using a formula (i.e., by isolating the variable and then substituting known values; by substituting known values and then solving for the variable) (e.g., in analytic geometry, in measurement) (***Sample problem:*** A cone has a volume of 100 cm^3. The radius of the base is 3 cm. What is the height of the cone?);

– express the equation of a line in the form $y = mx + b$, given the form $Ax + By + C = 0$.

Graphing and Writing Equations of Lines

By the end of this course, students will:

– connect the rate of change of a linear relation to the slope of the line, and define the slope as the ratio $m = \frac{rise}{run}$;

– identify, through investigation, $y = mx + b$ as a common form for the equation of a straight line, and identify the special cases $x = a$, $y = b$;

– identify, through investigation with technology, the geometric significance of m and b in the equation $y = mx + b$;

– identify, through investigation, properties of the slopes of lines and line segments (e.g., direction, positive or negative rate of change, steepness, parallelism), using graphing technology to facilitate investigations, where appropriate;

– graph lines by hand, using a variety of techniques (e.g., graph $y = \frac{2}{3}x - 4$ using the y-intercept and slope; graph $2x + 3y = 6$ using the x- and y-intercepts);

– determine the equation of a line, given its graph, the slope and y-intercept, the slope and a point on the line, or two points on the line.

Solving and Interpreting Systems of Linear Equations

By the end of this course, students will:

– determine graphically the point of intersection of two linear relations (e.g., using graph paper, using technology) (***Sample problem:*** Determine the point of intersection of $y + 2x = -5$ and $y = \frac{2}{3}x + 3$, using an appropriate graphing technique, and verify.);

- solve systems of two linear equations involving two variables with integral coefficients, using the algebraic method of substitution or elimination (***Sample problem:*** Solve $y = 2x + 1$, $3x + 2y = 16$ for x and y algebraically, and verify algebraically and graphically.);
- solve problems that arise from realistic situations described in words or represented by given linear systems of two equations involving two variables, by choosing an appropriate algebraic or graphical method (***Sample problem:*** Maria has been hired by Company A with an annual salary, S dollars, given by $S = 32\ 500 + 500a$, where a represents the number of years she has been employed by this company. Ruth has been hired by Company B with an annual salary, S dollars, given by $S = 28\ 000 + 1000a$, where a represents the number of years she has been employed by that company. Describe what the solution of this system would represent in terms of Maria's salary and Ruth's salary. After how many years will their salaries be the same? What will their salaries be at that time?).

Quadratic Relations of the Form $y = ax^2 + bx + c$

Overall Expectations

By the end of this course, students will:

- manipulate algebraic expressions, as needed to understand quadratic relations;
- identify characteristics of quadratic relations;
- solve problems by interpreting graphs of quadratic relations.

Specific Expectations

Manipulating Quadratic Expressions

By the end of this course, students will:

– expand and simplify second-degree polynomial expressions involving one variable that consist of the product of two binomials [e.g., $(2x + 3)(x + 4)$] or the square of a binomial [e.g., $(x + 3)^2$], using a variety of tools (e.g., algebra tiles, diagrams, computer algebra systems, paper and pencil) and strategies (e.g. patterning);

– factor binomials (e.g., $4x^2 + 8x$) and trinomials (e.g., $3x^2 + 9x - 15$) involving one variable up to degree two, by determining a common factor using a variety of tools (e.g., algebra tiles, computer algebra systems, paper and pencil) and strategies (e.g., patterning);

– factor simple trinomials of the form $x^2 + bx + c$ (e.g., $x^2 + 7x + 10$, $x^2 + 2x - 8$), using a variety of tools (e.g., algebra tiles, computer algebra systems, paper and pencil) and strategies (e.g., patterning);

– factor the difference of squares of the form $x^2 - a^2$ (e.g., $x^2 - 16$).

Identifying Characteristics of Quadratic Relations

By the end of this course, students will:

– collect data that can be represented as a quadratic relation, from experiments using appropriate equipment and technology (e.g., concrete materials, scientific probes, graphing calculators), or from secondary sources (e.g., the Internet, Statistics Canada); graph the data and draw a curve of best fit, if appropriate, with or without the use of technology (***Sample problem:*** Make a 1 m ramp that makes a 15° angle with the floor. Place a can 30 cm up the ramp. Record the time it takes for the can to roll to the bottom. Repeat by placing the can 40 cm, 50 cm, and 60 cm up the ramp, and so on. Graph the data and draw the curve of best fit.);

– determine, through investigation using technology, that a quadratic relation of the form $y = ax^2 + bx + c$ $(a \neq 0)$ can be graphically represented as a parabola, and determine that the table of values yields a constant second difference (***Sample problem:*** Graph the quadratic relation $y = x^2 - 4$, using technology. Observe the shape of the graph. Consider the corresponding table of values, and calculate the first and second differences. Repeat for a different quadratic relation. Describe your observations and make conclusions.);

– identify the key features of a graph of a parabola (i.e., the equation of the axis of symmetry, the coordinates of the vertex, the y-intercept, the zeros, and the maximum or minimum value), using a given graph or a graph generated with technology from its equation, and use the appropriate terminology to describe the features;

– compare, through investigation using technology, the graphical representations of a quadratic relation in the form $y = x^2 + bx + c$ and the same relation in the factored form $y = (x - r)(x - s)$ (i.e., the graphs are the same), and describe the connections between each algebraic representation and the graph [e.g., the y-intercept is c in the form $y = x^2 + bx + c$; the x-intercepts are r and s in the form $y = (x - r)(x - s)$] (***Sample problem:*** Use a graphing calculator to compare the graphs of $y = x^2 + 2x - 8$ and $y = (x + 4)(x - 2)$. In what way(s) are the equations related? What information about the graph can you identify by looking at each equation? Make some conclusions from your observations, and check your conclusions with a different quadratic equation.).

Solving Problems by Interpreting Graphs of Quadratic Relations

By the end of this course, students will:

– solve problems involving a quadratic relation by interpreting a given graph or a graph generated with technology from its equation (e.g., given an equation representing the height of a ball over elapsed time, use a graphing calculator or graphing software to graph the relation, and answer questions such as the following: What is the maximum height of the ball? After what length of time will the ball hit the ground? Over what time interval is the height of the ball greater than 3 m?);

– solve problems by interpreting the significance of the key features of graphs obtained by collecting experimental data involving quadratic relations (***Sample problem:*** Roll a can up a ramp. Using a motion detector and a graphing calculator, record the motion of the can until it returns to its starting position, graph the distance from the starting position versus time, and draw the curve of best fit. Interpret the meanings of the vertex and the intercepts in terms of the experiment. Predict how the graph would change if you gave the can a harder push. Test your prediction.).