Functions, Grade 11

University Preparation MCR3U

This course introduces the mathematical concept of the function by extending students' experiences with linear and quadratic relations. Students will investigate properties of discrete and continuous functions, including trigonometric and exponential functions; represent functions numerically, algebraically, and graphically; solve problems involving applications of functions; investigate inverse functions; and develop facility in determining equivalent algebraic expressions. Students will reason mathematically and communicate their thinking as they solve multi-step problems.

Prerequisite: Principles of Mathematics, Grade 10, Academic

MATHEMATICAL PROCESS EXPECTATIONS

The mathematical processes are to be integrated into student learning in all areas of this course.

Throughout this course, students will:

Problem Solving	• develop, select, apply, compare, and adapt a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;
Reasoning and Proving	• develop and apply reasoning skills (e.g., use of inductive reasoning, deductive reasoning, and counter-examples; construction of proofs) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;
Reflecting	• demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);
Selecting Tools and Computational Strategies	• select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;
Connecting	• make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);
Representing	• create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems;
Communicating	• communicate mathematical thinking orally, visually, and in writing, using precise mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

A. CHARACTERISTICS OF FUNCTIONS

OVERALL EXPECTATIONS

By the end of this course, students will:

1. demonstrate an understanding of functions, their representations, and their inverses, and make connections between the algebraic and graphical representations of functions using transformations;
2. determine the zeros and the maximum or minimum of a quadratic function, and solve problems involving quadratic functions, including problems arising from real-world applications;
3. demonstrate an understanding of equivalence as it relates to simplifying polynomial, radical, and rational expressions.

SPECIFIC EXPECTATIONS

1. Representing Functions

By the end of this course, students will:

1.1 explain the meaning of the term *function*, and distinguish a function from a relation that is not a function, through investigation of linear and quadratic relations using a variety of representations (i.e., tables of values, mapping diagrams, graphs, function machines, equations) and strategies (e.g., identifying a one-to-one or many-to-one mapping; using the vertical-line test)

Sample problem: Investigate, using numeric and graphical representations, whether the relation $x = y^2$ is a function, and justify your reasoning.

1.2 represent linear and quadratic functions using function notation, given their equations, tables of values, or graphs, and substitute into and evaluate functions [e.g., evaluate $f\left(\frac{1}{2}\right)$, given $f(x) = 2x^2 + 3x - 1$]

1.3 explain the meanings of the terms *domain* and *range*, through investigation using numeric, graphical, and algebraic representations of the functions $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, and $f(x) = \frac{1}{x}$; describe the domain and range of a function appropriately (e.g., for $y = x^2 + 1$, the domain is the set of real numbers, and the range is $y \geq 1$); and explain any restrictions on the domain and range in contexts arising from real-world applications

Sample problem: A quadratic function represents the relationship between the height of a ball and the time elapsed since the ball was thrown. What physical factors will restrict the domain and range of the quadratic function?

1.4 relate the process of determining the inverse of a function to their understanding of reverse processes (e.g., applying inverse operations)

1.5 determine the numeric or graphical representation of the inverse of a linear or quadratic function, given the numeric, graphical, or algebraic representation of the function, and make connections, through investigation using a variety of tools (e.g., graphing technology, Mira, tracing paper), between the graph of a function and the graph of its inverse (e.g., the graph of the inverse is the reflection of the graph of the function in the line $y = x$)

Sample problem: Given a graph and a table of values representing population over time, produce a table of values for the inverse and graph the inverse on a new set of axes.

1.6 determine, through investigation, the relationship between the domain and range of a function and the domain and range of the inverse relation, and determine whether or not the inverse relation is a function

Sample problem: Given the graph of $f(x) = x^2$, graph the inverse relation. Compare the domain and range of the function with the domain and range of the inverse relation, and investigate connections to the domain and range of the functions $g(x) = \sqrt{x}$ and $h(x) = -\sqrt{x}$.

1.7 determine, using function notation when appropriate, the algebraic representation of the inverse of a linear or quadratic function, given the algebraic representation of the function [e.g., $f(x) = (x - 2)^2 - 5$], and make connections, through investigation using a variety of tools (e.g., graphing technology, Mira, tracing paper), between the algebraic representations of a function and its inverse (e.g., the inverse of a linear function involves applying the inverse operations in the reverse order)

Sample problem: Given the equations of several linear functions, graph the functions and their inverses, determine the equations of the inverses, and look for patterns that connect the equation of each linear function with the equation of the inverse.

1.8 determine, through investigation using technology, the roles of the parameters a, k, d, and c in functions of the form $y = af(k(x - d)) + c$, and describe these roles in terms of transformations on the graphs of $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, and $f(x) = \frac{1}{x}$ (i.e., translations; reflections in the axes; vertical and horizontal stretches and compressions to and from the x- and y-axes)

Sample problem: Investigate the graph $f(x) = 3(x - d)^2 + 5$ for various values of d, using technology, and describe the effects of changing d in terms of a transformation.

1.9 sketch graphs of $y = af(k(x - d)) + c$ by applying one or more transformations to the graphs of $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, and $f(x) = \frac{1}{x}$, and state the domain and range of the transformed functions

Sample problem: Transform the graph of $f(x)$ to sketch $g(x)$, and state the domain and range of each function, for the following: $f(x) = \sqrt{x}$, $g(x) = \sqrt{x - 4}$; $f(x) = \frac{1}{x}$, $g(x) = -\frac{1}{x + 1}$.

2. Solving Problems Involving Quadratic Functions

By the end of this course, students will:

2.1 determine the number of zeros (i.e., x-intercepts) of a quadratic function, using a variety of strategies (e.g., inspecting graphs; factoring; calculating the discriminant)

Sample problem: Investigate, using graphing technology and algebraic techniques, the transformations that affect the number of zeros for a given quadratic function.

2.2 determine the maximum or minimum value of a quadratic function whose equation is given in the form $f(x) = ax^2 + bx + c$, using an algebraic method (e.g., completing the square; factoring to determine the zeros and averaging the zeros)

Sample problem: Explain how partially factoring $f(x) = 3x^2 - 6x + 5$ into the form $f(x) = 3x(x - 2) + 5$ helps you determine the minimum of the function.

2.3 solve problems involving quadratic functions arising from real-world applications and represented using function notation

Sample problem: The profit, $P(x)$, of a video company, in thousands of dollars, is given by $P(x) = -5x^2 + 550x - 5000$, where x is the amount spent on advertising, in thousands of dollars. Determine the maximum profit that the company can make, and the amounts spent on advertising that will result in a profit and that will result in a profit of at least \$4 000 000.

2.4 determine, through investigation, the transformational relationship among the family of quadratic functions that have the same zeros, and determine the algebraic representation of a quadratic function, given the real roots of the corresponding quadratic equation and a point on the function

Sample problem: Determine the equation of the quadratic function that passes through (2, 5) if the roots of the corresponding quadratic equation are $1 + \sqrt{5}$ and $1 - \sqrt{5}$.

2.5 solve problems involving the intersection of a linear function and a quadratic function graphically and algebraically (e.g., determine the time when two identical cylindrical water tanks contain equal volumes of water, if one tank is being filled at a constant rate and the other is being emptied through a hole in the bottom)

Sample problem: Determine, through investigation, the equations of the lines that have a slope of 2 and that intersect the quadratic function $f(x) = x(6 - x)$ once; twice; never.

3. Determining Equivalent Algebraic Expressions*

By the end of this course, students will:

3.1 simplify polynomial expressions by adding, subtracting, and multiplying

Sample problem: Write and simplify an expression for the volume of a cube with edge length $2x + 1$.

3.2 verify, through investigation with and without technology, that $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$, $a \geq 0$, $b \geq 0$, and use this relationship to simplify radicals (e.g., $\sqrt{24}$) and radical expressions obtained by adding, subtracting, and multiplying [e.g., $(2 + \sqrt{6})(3 - \sqrt{12})$]

3.3 simplify rational expressions by adding, subtracting, multiplying, and dividing, and state the restrictions on the variable values

Sample problem: Simplify $\frac{2x}{4x^2 + 6x} - \frac{3}{2x + 3}$, and state the restrictions on the variable.

3.4 determine if two given algebraic expressions are equivalent (i.e., by simplifying; by substituting values)

Sample problem: Determine if the expressions $\frac{2x^2 - 4x - 6}{x + 1}$ and $8x^2 - 2x(4x - 1) - 6$ are equivalent.

*The knowledge and skills described in the expectations in this section are to be introduced as needed, and applied and consolidated, as appropriate, in solving problems throughout the course.

B. EXPONENTIAL FUNCTIONS

OVERALL EXPECTATIONS

By the end of this course, students will:

1. evaluate powers with rational exponents, simplify expressions containing exponents, and describe properties of exponential functions represented in a variety of ways;
2. make connections between the numeric, graphical, and algebraic representations of exponential functions;
3. identify and represent exponential functions, and solve problems involving exponential functions, including problems arising from real-world applications.

SPECIFIC EXPECTATIONS

1. Representing Exponential Functions

By the end of this course, students will:

1.1 graph, with and without technology, an exponential relation, given its equation in the form $y = a^x$ ($a > 0$, $a \neq 1$), define this relation as the function $f(x) = a^x$, and explain why it is a function

1.2 determine, through investigation using a variety of tools (e.g., calculator, paper and pencil, graphing technology) and strategies (e.g., patterning; finding values from a graph; interpreting the exponent laws), the value of a power with a rational exponent (i.e., $x^{\frac{m}{n}}$, where $x > 0$ and m and n are integers)

Sample problem: The exponent laws suggest that $4^{\frac{1}{2}} \times 4^{\frac{1}{2}} = 4^1$. What value would you assign to $4^{\frac{1}{2}}$? What value would you assign to $27^{\frac{1}{3}}$? Explain your reasoning. Extend your reasoning to make a generalization about the meaning of $x^{\frac{1}{n}}$, where $x > 0$ and n is a natural number.

1.3 simplify algebraic expressions containing integer and rational exponents [e.g., $(x^3) \div \left(x^{\frac{1}{2}}\right)$, $(x^6y^3)^{\frac{1}{3}}$], and evaluate numeric expressions containing integer and rational exponents and rational bases [e.g., 2^{-3}, $(-6)^3$, $4^{\frac{1}{2}}$, 1.01^{120}]

1.4 determine, through investigation, and describe key properties relating to domain and range, intercepts, increasing/decreasing intervals, and asymptotes (e.g., the domain is the set of real numbers; the range is the set of positive real numbers; the function either increases or decreases throughout its domain) for exponential functions represented in a variety of ways [e.g., tables of values, mapping diagrams, graphs, equations of the form $f(x) = a^x$ ($a > 0$, $a \neq 1$), function machines]

Sample problem: Graph $f(x) = 2^x$, $g(x) = 3^x$, and $h(x) = 0.5^x$ on the same set of axes. Make comparisons between the graphs, and explain the relationship between the y-intercepts.

2. Connecting Graphs and Equations of Exponential Functions

By the end of this course, students will:

2.1 distinguish exponential functions from linear and quadratic functions by making comparisons in a variety of ways (e.g., comparing rates of change using finite differences in tables of values; identifying a constant ratio in a table of values; inspecting graphs; comparing equations)

Sample problem: Explain in a variety of ways how you can distinguish the exponential function $f(x) = 2^x$ from the quadratic function $f(x) = x^2$ and the linear function $f(x) = 2x$.

2.2 determine, through investigation using technology, the roles of the parameters a, k, d, and c in functions of the form $y = af(k(x - d)) + c$, and describe these roles in terms of transformations on the graph of $f(x) = a^x$ $(a > 0, a \neq 1)$ (i.e., translations; reflections in the axes; vertical and horizontal stretches and compressions to and from the x- and y-axes)

Sample problem: Investigate the graph of $f(x) = 3^{x - d} - 5$ for various values of d, using technology, and describe the effects of changing d in terms of a transformation.

2.3 sketch graphs of $y = af(k(x - d)) + c$ by applying one or more transformations to the graph of $f(x) = a^x$ $(a > 0, a \neq 1)$, and state the domain and range of the transformed functions

Sample problem: Transform the graph of $f(x) = 3^x$ to sketch $g(x) = 3^{-(x + 1)} - 2$, and state the domain and range of each function.

2.4 determine, through investigation using technology, that the equation of a given exponential function can be expressed using different bases [e.g., $f(x) = 9^x$ can be expressed as $f(x) = 3^{2x}$], and explain the connections between the equivalent forms in a variety of ways (e.g., comparing graphs; using transformations; using the exponent laws)

2.5 represent an exponential function with an equation, given its graph or its properties

Sample problem: Write two equations to represent the same exponential function with a y-intercept of 5 and an asymptote at $y = 3$. Investigate whether other exponential functions have the same properties. Use transformations to explain your observations.

3. Solving Problems Involving Exponential Functions

By the end of this course, students will:

3.1 collect data that can be modelled as an exponential function, through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials such as number cubes, coins; measurement tools such as electronic probes), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data

Sample problem: Collect data and graph the cooling curve representing the relationship between temperature and time for hot water cooling in a porcelain mug. Predict the shape of the cooling curve when hot water cools in an insulated mug. Test your prediction.

3.2 identify exponential functions, including those that arise from real-world applications involving growth and decay (e.g., radioactive decay, population growth, cooling rates, pressure in a leaking tire), given various representations (i.e., tables of values, graphs, equations), and explain any restrictions that the context places on the domain and range (e.g., ambient temperature limits the range for a cooling curve)

Sample problem: Using data from Statistics Canada, investigate to determine if there was a period of time over which the increase in Canada's national debt could be modelled using an exponential function.

3.3 solve problems using given graphs or equations of exponential functions arising from a variety of real-world applications (e.g., radioactive decay, population growth, height of a bouncing ball, compound interest) by interpreting the graphs or by substituting values for the exponent into the equations

Sample problem: The temperature of a cooling liquid over time can be modelled by the exponential function

$T(x) = 60\left(\frac{1}{2}\right)^{\frac{x}{30}} + 20$, where $T(x)$ is the temperature, in degrees Celsius, and x is the elapsed time, in minutes. Graph the function and determine how long it takes for the temperature to reach 28ºC.

C. DISCRETE FUNCTIONS

OVERALL EXPECTATIONS

By the end of this course, students will:

1. demonstrate an understanding of recursive sequences, represent recursive sequences in a variety of ways, and make connections to Pascal's triangle;
2. demonstrate an understanding of the relationships involved in arithmetic and geometric sequences and series, and solve related problems;
3. make connections between sequences, series, and financial applications, and solve problems involving compound interest and ordinary annuities.

SPECIFIC EXPECTATIONS

1. Representing Sequences

By the end of this course, students will:

1.1 make connections between sequences and discrete functions, represent sequences using function notation, and distinguish between a discrete function and a continuous function [e.g., $f(x) = 2x$, where the domain is the set of natural numbers, is a discrete linear function and its graph is a set of equally spaced points; $f(x) = 2x$, where the domain is the set of real numbers, is a continuous linear function and its graph is a straight line]

1.2 determine and describe (e.g., in words; using flow charts) a recursive procedure for generating a sequence, given the initial terms (e.g., 1, 3, 6, 10, 15, 21, …), and represent sequences as discrete functions in a variety of ways (e.g., tables of values, graphs)

1.3 connect the formula for the nth term of a sequence to the representation in function notation, and write terms of a sequence given one of these representations or a recursion formula

1.4 represent a sequence algebraically using a recursion formula, function notation, or the formula for the nth term [e.g., represent 2, 4, 8, 16, 32, 64, … as $t_1 = 2$; $t_n = 2t_{n-1}$, as $f(n) = 2^n$, or as $t_n = 2^n$, or represent $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \ldots$ as $t_1 = \frac{1}{2}$; $t_n = t_{n-1} + \frac{1}{n(n+1)}$, as $f(n) = \frac{n}{n+1}$, or as $t_n = \frac{n}{n+1}$, where n is a natural number], and describe the information that can be obtained by inspecting each representation (e.g., function notation or the formula for the nth term may show the type of function; a recursion formula shows the relationship between terms)

Sample problem: Represent the sequence 0, 3, 8, 15, 24, 35, … using a recursion formula, function notation, and the formula for the nth term. Explain why this sequence can be described as a discrete quadratic function. Explore how to identify a sequence as a discrete quadratic function by inspecting the recursion formula.

1.5 determine, through investigation, recursive patterns in the Fibonacci sequence, in related sequences, and in Pascal's triangle, and represent the patterns in a variety of ways (e.g., tables of values, algebraic notation)

1.6 determine, through investigation, and describe the relationship between Pascal's triangle and the expansion of binomials, and apply the relationship to expand binomials raised to whole-number exponents [e.g., $(1 + x)^4$, $(2x - 1)^5$, $(2x - y)^6$, $(x^2 + 1)^5$]

2. Investigating Arithmetic and Geometric Sequences and Series

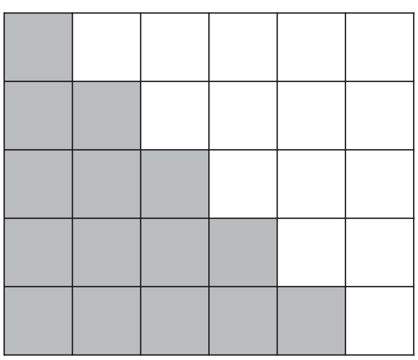
By the end of this course, students will:

2.1 identify sequences as arithmetic, geometric, or neither, given a numeric or algebraic representation

2.2 determine the formula for the general term of an arithmetic sequence [i.e., $t_n = a + (n - 1)d$] or geometric sequence (i.e., $t_n = ar^{n-1}$), through investigation using a variety of tools (e.g., linking cubes, algebra tiles, diagrams, calculators) and strategies (e.g., patterning; connecting the steps in a numerical example to the steps in the algebraic development), and apply the formula to calculate any term in a sequence

2.3 determine the formula for the sum of an arithmetic or geometric series, through investigation using a variety of tools (e.g., linking cubes, algebra tiles, diagrams, calculators) and strategies (e.g., patterning; connecting the steps in a numerical example to the steps in the algebraic development), and apply the formula to calculate the sum of a given number of consecutive terms

Sample problem: Given the following array built with grey and white connecting cubes, investigate how different ways of determining the total number of grey cubes can be used to evaluate the sum of the arithmetic series 1 + 2 + 3 + 4 + 5. Extend the series, use patterning to make generalizations for finding the sum, and test the generalizations for other arithmetic series.



2.4 solve problems involving arithmetic and geometric sequences and series, including those arising from real-world applications

3. Solving Problems Involving Financial Applications

By the end of this course, students will:

3.1 make and describe connections between simple interest, arithmetic sequences, and linear growth, through investigation with technology (e.g., use a spreadsheet or graphing calculator to make simple interest calculations, determine first differences in the amounts over time, and graph amount versus time)

Sample problem: Describe an investment that could be represented by the function $f(x) = 500(1 + 0.05x)$.

3.2 make and describe connections between compound interest, geometric sequences, and exponential growth, through investigation with technology (e.g., use a spreadsheet to make compound interest calculations, determine finite differences in the amounts over time, and graph amount versus time)

Sample problem: Describe an investment that could be represented by the function $f(x) = 500(1.05)^x$.

3.3 solve problems, using a scientific calculator, that involve the calculation of the amount, A (also referred to as future value, FV), the principal, P (also referred to as present value, PV), or the interest rate per compounding period, i, using the compound interest formula in the form $A = P(1 + i)^n$ [or $FV = PV(1 + i)^n$]

Sample problem: Two investments are available, one at 6% compounded annually and the other at 6% compounded monthly. Investigate graphically the growth of each investment, and determine the interest earned from depositing $1000 in each investment for 10 years.

3.4 determine, through investigation using technology (e.g., scientific calculator, the TVM Solver on a graphing calculator, online tools), the number of compounding periods, n, using the compound interest formula in the form $A = P(1 + i)^n$ [or $FV = PV(1 + i)^n$]; describe strategies (e.g., guessing and checking; using the power of a power rule for exponents; using graphs) for calculating this number; and solve related problems

3.5 explain the meaning of the term *annuity*, and determine the relationships between ordinary simple annuities (i.e., annuities in which payments are made at the *end* of each period, and compounding and payment periods are the same), geometric series, and exponential growth, through investigation with technology (e.g., use a spreadsheet to determine and graph the future value of an ordinary simple annuity for varying numbers of compounding periods; investigate how the contributions of each payment to the future value of an ordinary simple annuity are related to the terms of a geometric series)

3.6 determine, through investigation using technology (e.g., the TVM Solver on a graphing calculator, online tools), the effects of changing the conditions (i.e., the payments, the frequency of the payments, the interest rate, the compounding period) of ordinary simple annuities (e.g., long-term savings plans, loans)

Sample problem: Compare the amounts at age 65 that would result from making an annual deposit of $1000 starting at age 20, or from making an annual deposit of $3000 starting at age 50, to an RRSP that earns 6% interest per annum, compounded annually. What is the total of the deposits in each situation?

3.7 solve problems, using technology (e.g., scientific calculator, spreadsheet, graphing calculator), that involve the amount, the present value, and the regular payment of an ordinary simple annuity (e.g., calculate the total interest paid over the life of a loan, using a spreadsheet, and compare the total interest with the original principal of the loan)

D. TRIGONOMETRIC FUNCTIONS

OVERALL EXPECTATIONS

By the end of this course, students will:

1. determine the values of the trigonometric ratios for angles less than 360º; prove simple trigonometric identities; and solve problems using the primary trigonometric ratios, the sine law, and the cosine law;
2. demonstrate an understanding of periodic relationships and sinusoidal functions, and make connections between the numeric, graphical, and algebraic representations of sinusoidal functions;
3. identify and represent sinusoidal functions, and solve problems involving sinusoidal functions, including problems arising from real-world applications.

SPECIFIC EXPECTATIONS

1. Determining and Applying Trigonometric Ratios

By the end of this course, students will:

1.1 determine the exact values of the sine, cosine, and tangent of the special angles: 0º, 30º, 45º, 60º, and 90º

1.2 determine the values of the sine, cosine, and tangent of angles from 0º to 360º, through investigation using a variety of tools (e.g., dynamic geometry software, graphing tools) and strategies (e.g., applying the unit circle; examining angles related to special angles)

1.3 determine the measures of two angles from 0º to 360º for which the value of a given trigonometric ratio is the same

1.4 define the secant, cosecant, and cotangent ratios for angles in a right triangle in terms of the sides of the triangle (e.g., $\sec A = \frac{hypotenuse}{adjacent}$), and relate these ratios to the cosine, sine, and tangent ratios (e.g., $\sec A = \frac{1}{\cos A}$)

1.5 prove simple trigonometric identities, using the Pythagorean identity $\sin^2 x + \cos^2 x = 1$; the quotient identity $\tan x = \frac{\sin x}{\cos x}$; and the reciprocal identities $\sec x = \frac{1}{\cos x}$, $\csc x = \frac{1}{\sin x}$, and $\cot x = \frac{1}{\tan x}$

Sample problem: Prove that $1 - \cos^2 x = \sin x \cos x \tan x$.

1.6 pose problems involving right triangles and oblique triangles in two-dimensional settings, and solve these and other such problems using the primary trigonometric ratios, the cosine law, and the sine law (including the ambiguous case)

1.7 pose problems involving right triangles and oblique triangles in three-dimensional settings, and solve these and other such problems using the primary trigonometric ratios, the cosine law, and the sine law

Sample problem: Explain how a surveyor could find the height of a vertical cliff that is on the other side of a raging river, using a measuring tape, a theodolite, and some trigonometry. Determine what the surveyor might measure, and use hypothetical values for these data to calculate the height of the cliff.

2. Connecting Graphs and Equations of Sinusoidal Functions

By the end of this course, students will:

2.1 describe key properties (e.g., cycle, amplitude, period) of periodic functions arising from real-world applications (e.g., natural gas consumption in Ontario, tides in the Bay of Fundy), given a numeric or graphical representation

2.2 predict, by extrapolating, the future behaviour of a relationship modelled using a numeric or graphical representation of a periodic function (e.g., predicting hours of daylight on a particular date from previous measurements; predicting natural gas consumption in Ontario from previous consumption)

2.3 make connections between the sine ratio and the sine function and between the cosine ratio and the cosine function by graphing the relationship between angles from 0º to 360º and the corresponding sine ratios or cosine ratios, with or without technology (e.g., by generating a table of values using a calculator; by unwrapping the unit circle), defining this relationship as the function $f(x) = \sin x$ or $f(x) = \cos x$, and explaining why the relationship is a function

2.4 sketch the graphs of $f(x) = \sin x$ and $f(x) = \cos x$ for angle measures expressed in degrees, and determine and describe their key properties (i.e., cycle, domain, range, intercepts, amplitude, period, maximum and minimum values, increasing/decreasing intervals)

2.5 determine, through investigation using technology, the roles of the parameters a, k, d, and c in functions of the form $y = af(k(x - d)) + c$, where $f(x) = \sin x$ or $f(x) = \cos x$ with angles expressed in degrees, and describe these roles in terms of transformations on the graphs of $f(x) = \sin x$ and $f(x) = \cos x$ (i.e., translations; reflections in the axes; vertical and horizontal stretches and compressions to and from the x- and y-axes)

Sample problem: Investigate the graph $f(x) = 2\sin(x - d) + 10$ for various values of d, using technology, and describe the effects of changing d in terms of a transformation.

2.6 determine the amplitude, period, phase shift, domain, and range of sinusoidal functions whose equations are given in the form $f(x) = a\sin(k(x - d)) + c$ or $f(x) = a\cos(k(x - d)) + c$

2.7 sketch graphs of $y = af(k(x - d)) + c$ by applying one or more transformations to the graphs of $f(x) = \sin x$ and $f(x) = \cos x$, and state the domain and range of the transformed functions

Sample problem: Transform the graph of $f(x) = \cos x$ to sketch $g(x) = 3\cos 2x - 1$, and state the domain and range of each function.

2.8 represent a sinusoidal function with an equation, given its graph or its properties

Sample problem: A sinusoidal function has an amplitude of 2 units, a period of 180º, and a maximum at (0, 3). Represent the function with an equation in two different ways.

3. Solving Problems Involving Sinusoidal Functions

By the end of this course, students will:

3.1 collect data that can be modelled as a sinusoidal function (e.g., voltage in an AC circuit, sound waves), through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials, measurement tools such as motion sensors), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data

Sample problem: Measure and record distance−time data for a swinging pendulum, using a motion sensor or other measurement tools, and graph the data.

3.2 identify periodic and sinusoidal functions, including those that arise from real-world applications involving periodic phenomena, given various representations (i.e., tables of values, graphs, equations), and explain any restrictions that the context places on the domain and range

Sample problem: Using data from Statistics Canada, investigate to determine if there was a period of time over which changes in the population of Canadians aged 20–24 could be modelled using a sinusoidal function.

3.3 determine, through investigation, how sinusoidal functions can be used to model periodic phenomena that do not involve angles

Sample problem: Investigate, using graphing technology in degree mode, and explain how the function $h(t) = 5\sin(30(t + 3))$ approximately models the relationship between the height and the time of day for a tide with an amplitude of 5 m, if high tide is at midnight.

3.4 predict the effects on a mathematical model (i.e., graph, equation) of an application involving periodic phenomena when the conditions in the application are varied (e.g., varying the conditions, such as speed and direction, when walking in a circle in front of a motion sensor)

Sample problem: The relationship between the height above the ground of a person riding a Ferris wheel and time can be modelled using a sinusoidal function. Describe the effect on this function if the platform from which the person enters the ride is raised by 1 m and if the Ferris wheel turns twice as fast.

Sample problem: The height above the ground of a rider on a Ferris wheel can be modelled by the sinusoidal function $h(t) = 25\sin(3(t - 30)) + 27$, where $h(t)$ is the height, in metres, and t is the time, in seconds. Graph the function, using graphing technology in degree mode, and determine the maximum and minimum heights of the rider, the height after 30 s, and the time required to complete one revolution.

3.5 pose problems based on applications involving a sinusoidal function, and solve these and other such problems by using a given graph or a graph generated with technology from a table of values or from its equation