MPM2D: Principles of Mathematics

Using Trigonometric Ratios

Part 1: Solving For Unknown Sides

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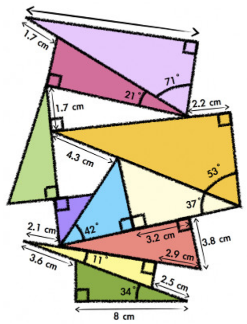
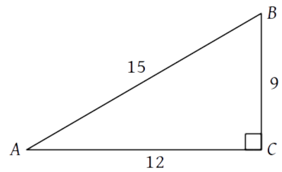


Primary Trigonometric Ratios

Recap

State the three primary trigonometric ratios for $\angle A$ in ΔABC.



$$\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{3}{5}$$

$$\cos A = \frac{\text{adj}}{\text{hyp}} = \frac{4}{5}$$

$$\tan A = \frac{\text{opp}}{\text{adj}} = \frac{3}{4}$$

Primary Trigonometric Ratios

Recall that a specific trigonometric ratio corresponds to a unique angle measurement.

For example, a 1° angle corresponds to a sine ratio of approximately 0.0175.

In the past, people used tables of trigonometric ratios that had been previously calculated by hand or computer, but nowadays any scientific calculator can calculate these ratios in a fraction of a second.

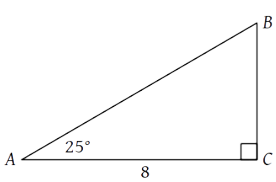
Primary Trigonometric Ratios

degrees	sine	cosine	tangent
1	0.0175	0.9998	0.0175
2	0.0349	0.9994	0.0349
3	0.0523	0.9986	0.0524
4	0.0698	0.9976	0.0699
5	0.0872	0.9962	0.0875
6	0.1045	0.9945	0.1051
7	0.1219	0.9925	0.1228
8	0.1392	0.9903	0.1405
⋮	⋮	⋮	⋮
88	0.9994	0.0349	28.6363
89	0.9998	0.0175	57.2900

Primary Trigonometric Ratios

Since the value a trigonometric ratio relates directly to a unique ratio of sides, it is possible to solve for an unknown side by using this value.

Consider the right triangle below.



How can we solve for $|BC|$?

Primary Trigonometric Ratios

We know that $\angle A = 25^\circ$, and that $|AC| = 8$. AC is adjacent to $\angle A$.

We want to know $|BC|$, which is opposite $\angle A$. Therefore, we can use the tangent ratio to relate the sides to the angle.

$$\tan A = \frac{\text{opp}}{\text{adj}}$$

$$\tan 25^\circ = \frac{|BC|}{8}$$

$$|BC| = 8 \tan 25^\circ$$

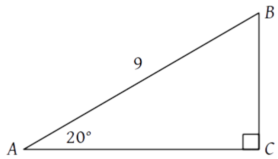
Using a calculator, $\tan 25^\circ \approx 0.4663$. Therefore, $|BC| \approx 8 \times 0.4663 \approx 3.73$.

It is important to choose the right ratio that relates the given and unknown sides.

Primary Trigonometric Ratios

Example

Determine $|AC|$.



Relative to $\angle A$, we know $|AB|$ (hypotenuse) and want to know $|AC|$ (adjacent). Use the cosine ratio to solve.

Primary Trigonometric Ratios

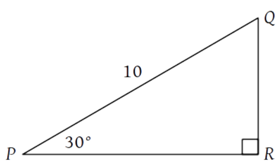
$$\begin{aligned}\cos A &= \frac{\text{adj}}{\text{hyp}}\\ \cos 20^\circ &= \frac{|AC|}{9}\\ |AC| &= 9\cos 20^\circ\\ |AC| &\approx 8.46\end{aligned}$$

In nearly all cases solutions will need to be rounded, since most trigonometric ratios are irrational numbers.

Primary Trigonometric Ratios

Example

Determine $|QR|$.



Relative to $\angle P$, we know $|PQ|$ (hypotenuse) and want to know $|QR|$ (opposite). Use the sine ratio to solve.

Primary Trigonometric Ratios

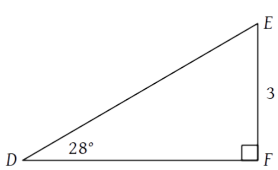
$$\begin{aligned}\sin P &= \frac{\text{opp}}{\text{hyp}}\\ \sin 30^\circ &= \frac{|QR|}{10}\\ |QR| &= 10\sin 30^\circ\\ |QR| &= 5\end{aligned}$$

In this case, $\sin 30^\circ = \frac{1}{2}$, so the solution is rational.

Primary Trigonometric Ratios

Example

Determine $|DF|$.



Relative to $\angle D$, we know $|EF|$ (opposite) and want to know $|DF|$ (adjacent). Use the tangent ratio to solve.

Primary Trigonometric Ratios

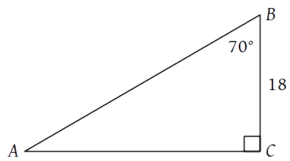
$$\begin{aligned}\tan D &= \frac{\text{opp}}{\text{adj}}\\ \tan 28^\circ &= \frac{3}{|DF|}\\ |DF|\tan 28^\circ &= 3\\ |DF| &= \frac{3}{\tan 28^\circ}\\ |DF| &\approx 5.64\end{aligned}$$

Note that the two-step process for isolating $|DF|$ above merely swaps the positions of $|DF|$ and $\tan D$.

Primary Trigonometric Ratios

Example

Determine $|AB|$.



Relative to $\angle B$, we know $|BC|$ (adjacent) and want to know $|AB|$ (hypotenuse). Use the cosine ratio to solve.

Primary Trigonometric Ratios

$$\cos B = \frac{\text{adj}}{\text{hyp}}$$
$$\cos 70^\circ = \frac{18}{|AB|}$$
$$|AB| = \frac{18}{\cos 70^\circ}$$
$$|AB| \approx 52.63$$

Questions?