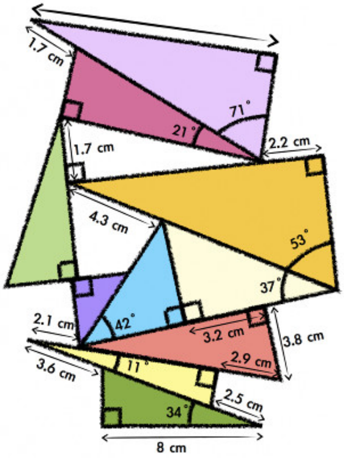
MPM2D: Principles of Mathematics

Using Trigonometric Ratios
Part 1: Solving For Unknown Sides

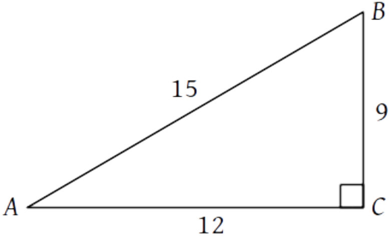
J. Garvin



Primary Trigonometric Ratios

Recap

State the three primary trigonometric ratios for $\angle A$ in ΔABC.



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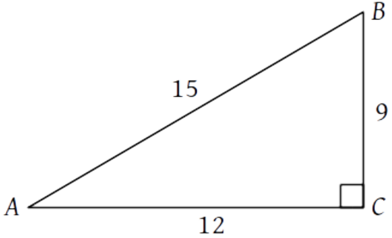

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$$= \frac{3}{5}$$

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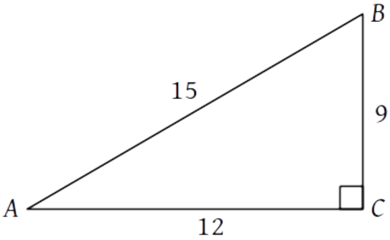

$$\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{3}{5}$$

$$\cos A = \frac{\text{adj}}{\text{hyp}} = \frac{4}{5}$$

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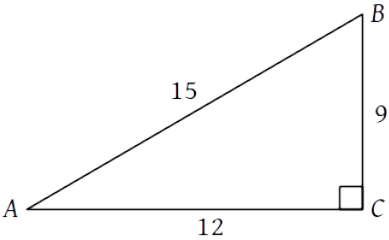

$$\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{3}{5}$$

$$\cos A = \frac{\text{adj}}{\text{hyp}} = \frac{4}{5}$$

$$\tan A = \frac{\text{opp}}{\text{adj}} = \frac{3}{4}$$

Primary Trigonometric Ratios

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In the past, people used tables of trigonometric ratios that had been previously calculated by hand or computer, but nowadays any scientific calculator can calculate these ratios in a fraction of a second.

Primary Trigonometric Ratios

degrees	sine	cosine	tangent
1	0.0175	0.9998	0.0175
2	0.0349	0.9994	0.0349
3	0.0523	0.9986	0.0524
4	0.0698	0.9976	0.0699
5	0.0872	0.9962	0.0875
6	0.1045	0.9945	0.1051
7	0.1219	0.9925	0.1228
8	0.1392	0.9903	0.1405
⋮	⋮	⋮	⋮
88	0.9994	0.0349	28.6363
89	0.9998	0.0175	57.2900

Primary Trigonometric Ratios

Since the value a trigonometric ratio relates directly to a unique ratio of sides, it is possible to solve for an unknown side by using this value.

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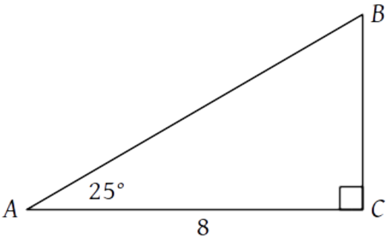
Consider the right triangle below.



How can we solve for $|BC|$?

Primary Trigonometric Ratios

We know that $\angle A = 25^\circ$, and that $|AC| = 8$. AC is adjacent to $\angle A$.

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$$|BC| = 8 \tan 25^\circ$$

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$$\begin{aligned} \tan A &= \frac{\text{opp}}{\text{adj}} \\ \tan 25^\circ &= \frac{|BC|}{8} \\ |BC| &= 8 \tan 25^\circ \end{aligned}$$

Using a calculator, $\tan 25^\circ \approx 0.4663$. Therefore, $|BC| \approx 8 \times 0.4663 \approx 3.73$.

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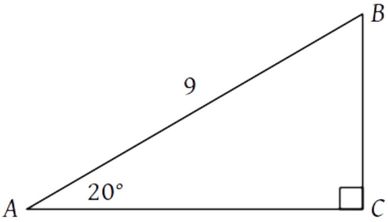
Using a calculator, $\tan 25^\circ \approx 0.4663$. Therefore, $|BC| \approx 8 \times 0.4663 \approx 3.73$.

It is important to choose the right ratio that relates the given and unknown sides.

Primary Trigonometric Ratios

Example

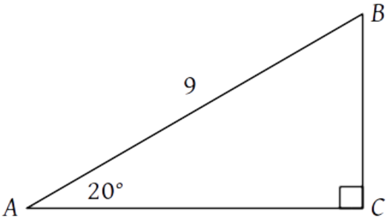
Determine $|AC|$.



Primary Trigonometric Ratios

Example

Determine $|AC|$.



Relative to $\angle A$, we know $|AB|$ (hypotenuse) and want to know $|AC|$ (adjacent). Use the cosine ratio to solve.

Primary Trigonometric Ratios

$$\cos A = \frac{\text{adj}}{\text{hyp}}$$

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Primary Trigonometric Ratios

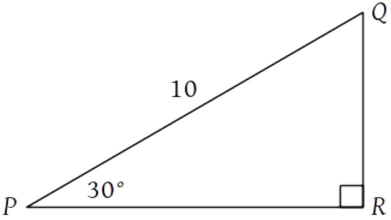
$$\begin{aligned}\cos A &= \frac{\text{adj}}{\text{hyp}} \\ \cos 20^\circ &= \frac{|AC|}{9} \\ |AC| &= 9\cos 20^\circ \\ |AC| &\approx 8.46\end{aligned}$$

In nearly all cases solutions will need to be rounded, since most trigonometric ratios are irrational numbers.

Primary Trigonometric Ratios

Example

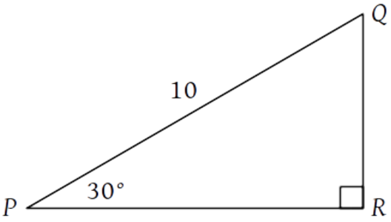
Determine $|QR|$.



Primary Trigonometric Ratios

Example

Determine $|QR|$.



Relative to $\angle P$, we know $|PQ|$ (hypotenuse) and want to know $|QR|$ (opposite). Use the sine ratio to solve.

Primary Trigonometric Ratios

$$\sin P = \frac{\text{opp}}{\text{hyp}}$$

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$$\sin P = \frac{\text{opp}}{\text{hyp}}$$
$$\sin 30^\circ = \frac{|QR|}{10}$$

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$$|QR| = 10 \sin 30^\circ$$

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$$\sin P = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 30^\circ = \frac{|QR|}{10}$$

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$$|QR| = 5$$

Primary Trigonometric Ratios

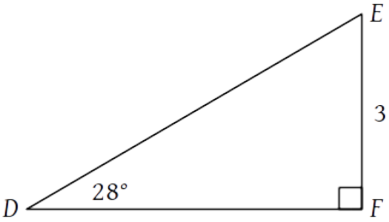
$$\begin{aligned} \sin P &= \frac{\text{opp}}{\text{hyp}} \\ \sin 30^\circ &= \frac{|QR|}{10} \\ |QR| &= 10 \sin 30^\circ \\ |QR| &= 5 \end{aligned}$$

In this case, $\sin 30^\circ = \frac{1}{2}$, so the solution is rational.

Primary Trigonometric Ratios

Example

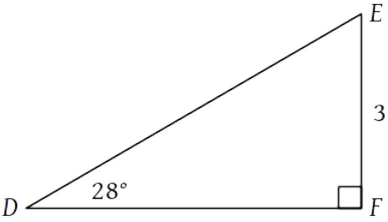
Determine $|DF|$.



Primary Trigonometric Ratios

Example

Determine $|DF|$.



Relative to $\angle D$, we know $|EF|$ (opposite) and want to know $|DF|$ (adjacent). Use the tangent ratio to solve.

Primary Trigonometric Ratios

$$\tan D = \frac{\text{opp}}{\text{adj}}$$

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$$\tan 28^\circ = \frac{3}{|DF|}$$

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$$\begin{aligned} \tan D &= \frac{\text{opp}}{\text{adj}} \\ \tan 28^\circ &= \frac{3}{|DF|} \\ |DF| \tan 28^\circ &= 3 \end{aligned}$$

Primary Trigonometric Ratios

$$\begin{aligned} \tan D &= \frac{\text{opp}}{\text{adj}} \\ \tan 28^\circ &= \frac{3}{|DF|} \\ |DF| \tan 28^\circ &= 3 \\ |DF| &= \frac{3}{\tan 28^\circ} \end{aligned}$$

Primary Trigonometric Ratios

$$\begin{aligned}
\tan D &= \frac{\text{opp}}{\text{adj}} \\
\tan 28^\circ &= \frac{3}{|DF|} \\
|DF| \tan 28^\circ &= 3 \\
|DF| &= \frac{3}{\tan 28^\circ} \\
|DF| &\approx 5.64
\end{aligned}$$

Primary Trigonometric Ratios

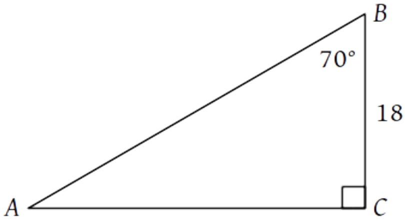
$$\tan D = \frac{\text{opp}}{\text{adj}}$$
$$\tan 28^\circ = \frac{3}{|DF|}$$
$$|DF| \tan 28^\circ = 3$$
$$|DF| = \frac{3}{\tan 28^\circ}$$
$$|DF| \approx 5.64$$

Note that the two-step process for isolating $|DF|$ above merely swaps the positions of $|DF|$ and $\tan D$.

Primary Trigonometric Ratios

Example

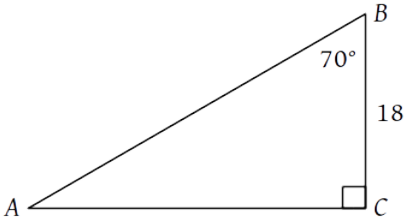
Determine $|AB|$.



Primary Trigonometric Ratios

Example

Determine $|AB|$.



Relative to $\angle B$, we know $|BC|$ (adjacent) and want to know $|AB|$ (hypotenuse). Use the cosine ratio to solve.

Primary Trigonometric Ratios

$$\cos B = \frac{\text{adj}}{\text{hyp}}$$

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$$\cos 70^\circ = \frac{18}{|AB|}$$

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$$|AB| = \frac{18}{\cos 70^\circ}$$

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$$\cos 70^\circ = \frac{18}{|AB|}$$

$$|AB| = \frac{18}{\cos 70^\circ}$$

$$|AB| \approx 52.63$$

Questions?