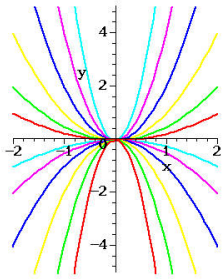
MPM2D: Principles of Mathematics

Quadratic Relations

Quadratics In Standard Form

J. Garvin



Linear Relations

Recall

Sketch a graph of $y = 2x - 6$ and discuss its key features.

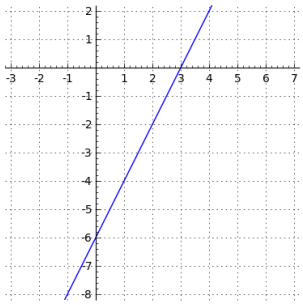
The relation is linear, with a constant rate of change (slope) of 2.

The graph is a straight line, with a y-intercept at $(0, 6)$ and an x-intercept at $(3, 0)$.

The line extends infinitely, and there is no minimum or maximum value.

Linear Relations

A graph of the relation is below.



Quadratic Relations

Not all relations are linear in nature.

The graph of a quadratic relation of the form $y = ax^2 + bx + c$ is not a straight line, since the value of x^2 increases more quickly as x gets larger.

What do the graphs of quadratic relations look like?

Quadratic Relations

Example

Sketch a graph of $y = x^2 - 6x + 5$.

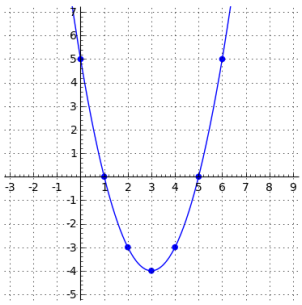
Construct a table of values to determine some points.

x	$y = x^2 - 6x + 5$
0	$0^2 - 6(0) + 5 = 5$
1	$1^2 - 6(1) + 5 = 0$
2	$2^2 - 6(2) + 5 = -3$
3	$3^2 - 6(3) + 5 = -4$
4	$4^2 - 6(4) + 5 = -3$
5	$5^2 - 6(5) + 5 = 0$
6	$6^2 - 6(6) + 5 = 5$

Note that there is symmetry in the values.

Quadratic Relations

A graph of the relation is below.



Quadratic Relations

The resulting shape is called a *parabola*.

Like a linear relation, a parabola has a y-intercept when $x = 0$. The parabola on the previous slide has a y-intercept at $(0, 5)$.

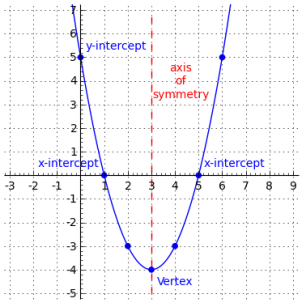
Unlike a linear relation, the curve of the parabola makes it possible to have up to 2 x-intercepts. In this case, there are x-intercepts at $(1, 0)$ and $(5, 0)$.

The *vertex* of a parabola is its lowest or its highest point. The previous parabola's vertex is at $(3, -4)$.

A parabola is symmetric in its *axis of symmetry*. The axis is a vertical line, passing through the vertex. In this case, the axis of symmetry has equation $x = 3$.

Quadratic Relations

These key features are summarized below.



Quadratic Relations

Example

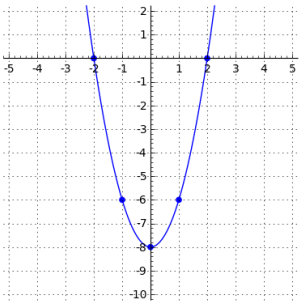
Sketch a graph of $y = 2x^2 - 8$ and state its key features.

Again, use a table of values to determine some points.

x	$y = 2x^2 - 8$
-2	$2(-2)^2 - 8 = 0$
-1	$2(-1)^2 - 8 = -6$
0	$2(0)^2 - 8 = -8$
1	$2(1)^2 - 8 = -6$
2	$2(2)^2 - 8 = 0$

Quadratic Relations

A graph of the relation is below.



Quadratic Relations

The key features of the graph of $y = 2x^2 - 8$ are:

- the parabola opens upward
- the y-intercept is at $(0, -8)$
- there are x-intercepts at $(-2, 0)$ and $(2, 0)$
- the vertex is a minimum, and is also the y-intercept
- the axis of symmetry is the y-axis, with equation $x = 0$

Quadratic Relations

Example

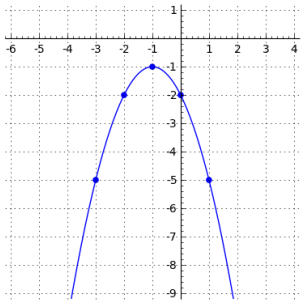
Sketch a graph of $y = -x^2 - 2x - 2$ and state its key features.

Again, use a table of values to determine some points.

x	$y = -x^2 - 2x - 2$
-3	$-(-3)^2 - 2(-3) - 2 = -5$
-2	$-(-2)^2 - 2(-2) - 2 = -2$
-1	$-(-1)^2 - 2(-1) - 2 = -1$
0	$-(0)^2 - 2(0) - 2 = -2$
1	$-(1)^2 - 2(1) - 2 = -5$

Quadratic Relations

A graph of the relation is below.



Quadratic Relations

The key features of the graph of $y = -x^2 - 2x - 2$ are:

- the parabola opens downward
- the y-intercept is at $(0, -2)$
- there are no x-intercepts
- the vertex is a maximum at $(-1, -1)$
- the axis of symmetry has equation $x = -1$

Quadratic Relations

Recall that the first *finite differences* of a linear relation are constant.

A quadratic relation, on the other hand, has constant second finite differences.

Compare the two relations below.

$y = 2x + 5$

x	y	
1	7	$\Delta 1$
2	9	2
3	11	2
4	13	2
5	15	2

$y = x^2 - 3$

x	y		
1	-2	$\Delta 1$	
2	1	3	$\Delta 2$
3	6	5	2
4	13	7	2
5	22	9	2

Quadratic Relations

Example

Classify the relation below as linear, quadratic or neither.

x	y		
1	-7	$\Delta 1$	
2	2	9	$\Delta 2$
3	17	15	6
4	38	21	6
5	65	27	6

Since the second finite differences are constant, the relation is quadratic.

Quadratic Relations

Example

Classify the relation below as linear, quadratic or neither.

x	y		
1	-4	$\Delta 1$	
2	3	7	$\Delta 2$
3	22	19	12
4	59	37	18
5	120	61	24

Since the neither the first nor second finite differences are constant, the relation is neither linear nor quadratic.

Questions?