MPM2D: Principles of Mathematics

Solving Quadratic Equations

Part 1: Solving by Factoring

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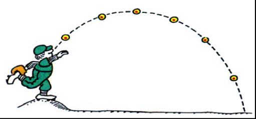


Quadratic Equations

If $a \cdot b = 0$, what can we say about a and b?

The only way in which two values can have a product of zero is if one of the values is zero.

Therefore, either $a = 0$ or $b = 0$.

What can be said about $a \cdot b = k$, if $k \neq 0$? For example, if $a \cdot b = 6$?

Not much can be said about either a or b, other than their signs must be the same (both positive or both negative), since there are many ways a and b can have a product of 6.

$2 \cdot 3 = 6$, $1 \cdot 6 = 6$, $(-1)(-6) = 6$, $\frac{1}{2}(12) = 6$, $\pi \cdot \frac{6}{\pi} = 6$, ...

Quadratic Equations

What about quadratic equations like $(x-1)(x-3) = 0$?

If $a = x - 1$ and $b = x - 3$, then we have $a \cdot b = 0$.

This means that either $x - 1 = 0$, or $x - 3 = 0$.

Solving each of these linear equations gives us two *solutions* to the quadratic relation: $x = 1$ or $x = 3$.

Testing either of these values shows that this is true.

$$(1-1)(1-3) = (0)(-2) = 0$$
$$(3-1)(3-3) = (2)(0) = 0$$

This means that if we can express a quadratic relation in factored form, such that the product of the two binomials is zero, then we can solve for the values of x.

Quadratic Equations

Example

Solve $x^2 - 4x - 12 = 0$.

Rewrite the simple trinomial as the product of two binomials.

$$(x-6)(x+2) = 0$$

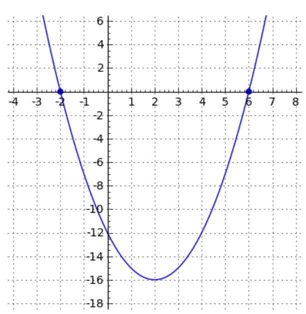
In order for the product to equal zero, one of the two binomials must equal zero.

$$x - 6 = 0 \qquad x + 2 = 0$$
$$x = 6 \qquad x = -2$$

Therefore, the two solutions are $x = 6$ and $x = -2$.

Note that the solutions (or *roots*, or *zeroes*) correspond to the x-intercepts of the parabola.

Quadratic Equations



Quadratic Equations

Example

Solve $x^2 + 8x = 20$.

Collect all terms on one side.

$$x^2 + 8x - 20 = 0$$

Factor the simple trinomial.

$$(x+10)(x-2) = 0$$

Equate each binomial with zero.

$$x + 10 = 0 \qquad x - 2 = 0$$
$$x = -10 \qquad x = 2$$

Therefore, the two solutions are $x = -10$ and $x = 2$.

Quadratic Equations

Example

Solve $2x^2 + 5x - 12 = 0$.

Use decomposition to break down the complex trinomial.

$$\begin{aligned} 2x^2 + 8x - 3x - 12 &= 0 \\ 2x(x+4) - 3(x+4) &= 0 \\ (x+4)(2x-3) &= 0 \end{aligned}$$

Equate each binomial with zero.

$$x + 4 = 0 \qquad 2x - 3 = 0$$
$$x = -4 \qquad x = \tfrac{3}{2}$$

Therefore, the two solutions are $x = -4$ and $x = \frac{3}{2}$.

The same solutions could also have been found by factoring the equation as $2(x+4)\left(x - \frac{3}{2}\right) = 0$.

Quadratic Equations

Example

Solve $3x^2 - 33x + 84 = 0$.

Common factor 3 from all terms.

$$3(x^2 - 11x + 28) = 0$$

Factor the simple trinomial, then divide both sides by 3.

$$\begin{aligned} 3(x-4)(x-7) &= 0 \\ (x-4)(x-7) &= 0 \end{aligned}$$

Equate each binomial with zero.

$$x - 4 = 0 \qquad x - 7 = 0$$
$$x = 4 \qquad x = 7$$

Therefore, the two solutions are $x = 4$ and $x = 7$.

Quadratic Equations

Example

Solve $4x^2 - 9 = 0$.

Factor the difference of squares.

$$\begin{aligned} 4x^2 - 9 &= 0 \\ (2x+3)(2x-3) &= 0 \end{aligned}$$

Equate each binomial with zero.

$$2x + 3 = 0 \qquad 2x - 3 = 0$$
$$x = -\tfrac{3}{2} \qquad x = \tfrac{3}{2}$$

Therefore, the two solutions are $x = -\frac{3}{2}$ and $x = \frac{3}{2}$.

Quadratic Equations

An alternative method for solving is to algebraically isolate x, instead of factoring.

$$\begin{aligned} 4x^2 - 9 &= 0 \\ 4x^2 &= 9 \\ x^2 &= \tfrac{9}{4} \\ x &= \pm\tfrac{3}{2} \end{aligned}$$

Remember that when removing a square, it is necessary to account for both a positive and a negative solution, unless there is a specific reason to reject one.

Quadratic Equations

Example

Solve $x^2 - 10x = -25$.

Collect all terms on one side.

$$x^2 - 10x + 25 = 0$$

Factor the perfect square trinomial.

$$(x-5)^2 = 0$$

Isolate x by taking the square root of both sides.

$$\begin{aligned} x - 5 &= 0 \\ x &= 5 \end{aligned}$$

In this case, 5 is the only solution to the equation.

Quadratic Equations

Example

Solve $2x^2 + 5x - 9 = 6 - 2x$.

Gather and collect like terms on one side.

$$2x^2 + 7x - 15 = 0$$

Use decomposition to factor the complex trinomial.

$$\begin{aligned} 2x^2 + 10x - 3x - 15 &= 0 \\ 2x(x+5) - 3(x+5) &= 0 \\ (x+5)(2x-3) &= 0 \end{aligned}$$

Equate the binomials with zero to find the solutions.

$$x + 5 = 0 \qquad 2x - 3 = 0$$
$$x = -5 \qquad x = \tfrac{3}{2}$$

Quadratic Equations

Example

Solve $4x^2 + 19x - 2 = 3x^2 + 5x + 7$.

Gather and collect like terms on one side.

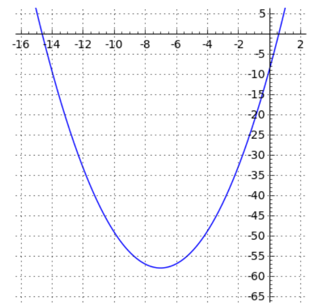
$$x^2 + 14x - 9 = 0$$

This equation does not factor, so it cannot be solved using the previous techniques.

A graph of the quadratic relation shows that the x-intercepts are not integral, and probably *irrational*.

We will develop a method for solving such equations in the next lesson.

Quadratic Equations



Questions?