MPM2D: Principles of Mathematics

The Sine Law

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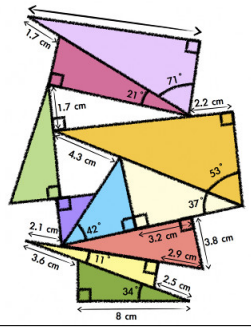
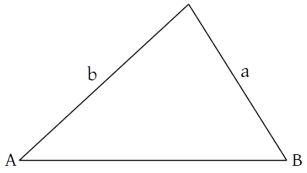


Sine Law

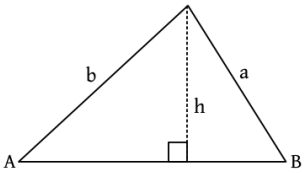
Consider the *oblique* triangle shown below, where $\angle A$, $\angle B$ and b are all known values.



How can we determine the length of a?

Sine Law

We can construct two right triangles, as shown.



In the leftmost triangle, $\sin A = \frac{h}{b}$, so $h = b \sin A$.

In the rightmost triangle, $\sin B = \frac{h}{a}$, so $h = a \sin B$.

Sine Law

Since $b \sin A$ and $a \sin B$ are both equal to h, they must be equal to each other.

Therefore, $b \sin A = a \sin B$, or $\frac{\sin A}{a} = \frac{\sin B}{b}$.

Law of Sines

Given ΔABC, $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$.

Note that each ratio involves the sine of an angle and its opposite side.

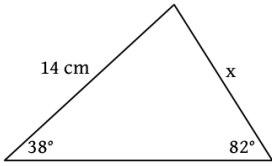
This relationship is true for any oblique triangle.

This relationship is also true for right triangles, but using the primary trigonometric ratios is preferred (and faster).

Sine Law

Example

Determine the length of x in the diagram below.



Sine Law

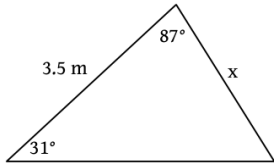
Use the Sine Law to solve for x.

$$\frac{\sin 82^\circ}{14} = \frac{\sin 38^\circ}{x}$$
$$x \sin 82^\circ = 14 \sin 38^\circ$$
$$x = \frac{14 \sin 38^\circ}{\sin 82^\circ}$$
$$x \approx 8.7 \text{ cm}$$

Sine Law

Example

Determine the length of x in the diagram below.



Sine Law

Since the Sine Law requires angles and their opposite sides, find the measure of the other angle first.

$$180^\circ - 87^\circ - 31^\circ = 62^\circ.$$

$$\frac{\sin 62^\circ}{3.5} = \frac{\sin 31^\circ}{x}$$

$$x \sin 62^\circ = 3.5 \sin 31^\circ$$

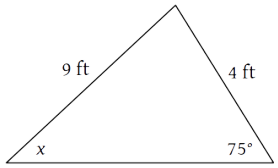
$$x = \frac{3.5 \sin 31^\circ}{\sin 62^\circ}$$

$$x \approx 2.04 \text{ m}$$

Sine Law

Example

Determine the measure of $\angle x$ in the diagram below.



Sine Law

Use the Sine Law to solve for $\angle x$.

$$\frac{\sin x}{4} = \frac{\sin 75^\circ}{9}$$

$$\sin x = \frac{4 \sin 75^\circ}{9}$$

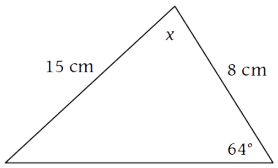
$$x = \sin^{-1}\left(\frac{4 \sin 75^\circ}{9}\right)$$

$$x \approx 25.4^\circ$$

Sine Law

Example

Determine the measure of $\angle x$ in the diagram below.



Sine Law

Use the Sine Law to solve for the other angle ($\angle y$) first, then determine $\angle x$.

$$\frac{\sin y}{8} = \frac{\sin 64^\circ}{15}$$

$$\sin y = \frac{8 \sin 64^\circ}{15}$$

$$y = \sin^{-1}\left(\frac{8 \sin 64^\circ}{15}\right)$$

$$y \approx 28.6^\circ$$

Therefore, $\angle x \approx 180^\circ - 64^\circ - 28.6^\circ \approx 87.4^\circ$.

Questions?