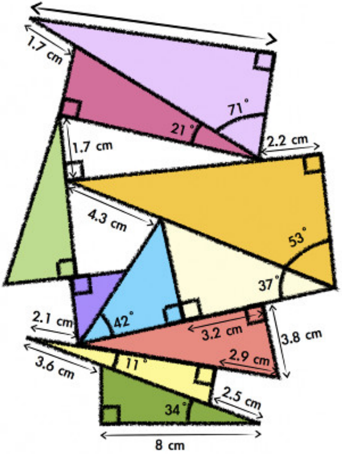
MPM2D: Principles of Mathematics

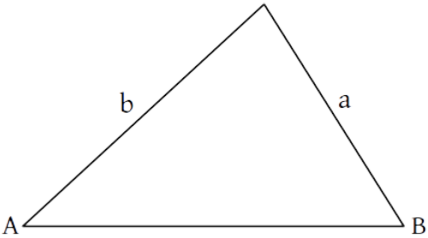
The Sine Law

J. Garvin



Sine Law

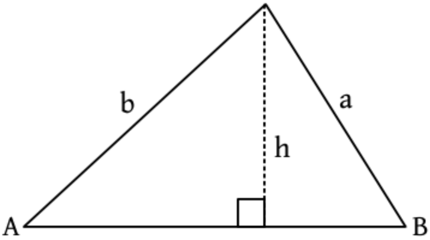
Consider the *oblique* triangle shown below, where $\angle A$, $\angle B$ and b are all known values.



How can we determine the length of a?

Sine Law

We can construct two right triangles, as shown.



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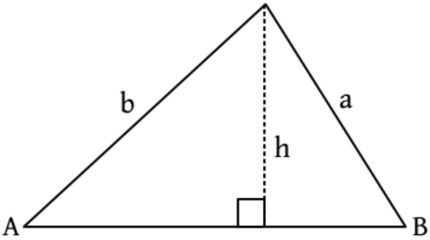

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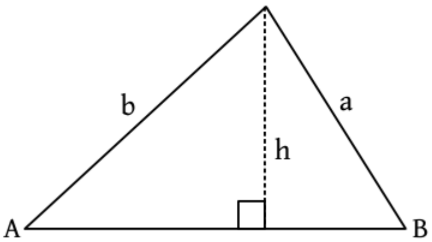

In the leftmost triangle, $\sin A = \frac{h}{b}$, so $h = b \sin A$.

In the rightmost triangle, $\sin B = \frac{h}{a}$, so $h = a \sin B$.

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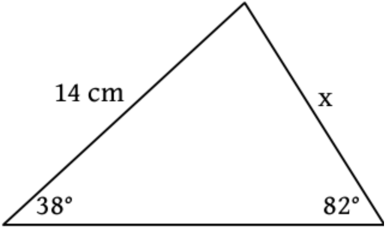
This relationship is also true for right triangles, but using the primary trigonometric ratios is preferred (and faster).

Sine Law

Example

Determine the length of x in the diagram below.



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$$\begin{aligned}\frac{\sin 82^\circ}{14} &= \frac{\sin 38^\circ}{x}\\ x\sin 82^\circ &= 14\sin 38^\circ\\ x &= \frac{14\sin 38^\circ}{\sin 82^\circ}\end{aligned}$$

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$$\begin{aligned} \frac{\sin 82^\circ}{14} &= \frac{\sin 38^\circ}{x} \\ x \sin 82^\circ &= 14 \sin 38^\circ \\ x &= \frac{14 \sin 38^\circ}{\sin 82^\circ} \\ x &\approx 8.7 \text{ cm} \end{aligned}$$

Sine Law

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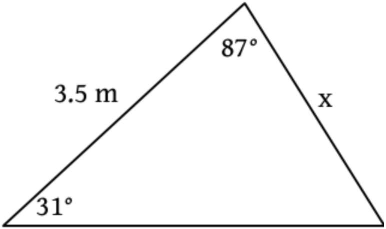
Determine the length of x in the diagram below.



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$$x \sin 62^\circ = 3.5 \sin 31^\circ$$

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$$x \approx 2.04 \text{ m}$$

Sine Law

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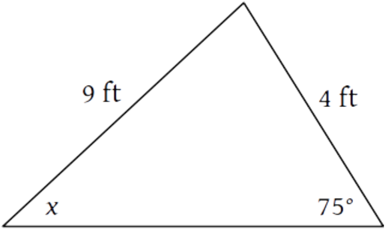
Determine the measure of $\angle x$ in the diagram below.



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$$x = \sin^{-1}\left(\frac{4 \sin 75^\circ}{9}\right)$$

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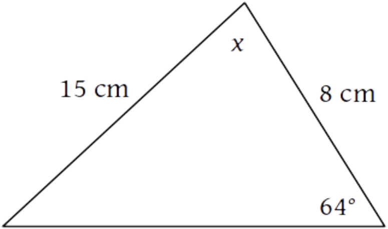
Use the Sine Law to solve for $\angle x$.

$$\begin{aligned} \frac{\sin x}{4} &= \frac{\sin 75^\circ}{9} \\ \sin x &= \frac{4 \sin 75^\circ}{9} \\ x &= \sin^{-1}\left(\frac{4 \sin 75^\circ}{9}\right) \\ x &\approx 25.4^\circ \end{aligned}$$

Sine Law

Example

Determine the measure of $\angle x$ in the diagram below.



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$$\begin{aligned}
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\sin y &= \frac{8 \sin 64^\circ}{15} \\
y &= \sin^{-1}\left(\frac{8 \sin 64^\circ}{15}\right) \\
y &\approx 28.6^\circ
\end{aligned}$$

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Use the Sine Law to solve for the other angle ($\angle y$) first, then determine $\angle x$.

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Therefore, $\angle x \approx 180^\circ - 64^\circ - 28.6^\circ \approx 87.4^\circ$.

Questions?