MPM2D: Principles of Mathematics

Simplifying Radicals

J. Garvin

Radicals

A *radical*, also called a *root*, is typically represented using the form $\sqrt[n]{x}$.

The argument x is called the *radicand*, while n is called the *index*.

For example, the third root (or *cube root*) of 5 is written $\sqrt[3]{5}$, and means "the value which, when multiplied by itself three times, gives five."

If no index is specified, the *square root* ($\sqrt[2]{x}$) is implied.

Mixed Radicals

A *mixed radical* is the product of a whole number and a radical.

For example, $2\sqrt{6}$ is the same as writing $2 \times \sqrt{6}$.

We often want to "simplify" radicals by writing them as mixed radicals, thus reducing the value of the radicand.

For example, $\sqrt{18}$ has the same value as $3\sqrt{2}$ (verify using a calculator), but the latter is in simplified form.

When a problem asks for a solution to use "exact values", keep any radicals rather than converting them to decimal approximations.

Simplifying Radicals

To simplify a radical such as $\sqrt{18}$, we use the following property of radicals.

Distributive Law of Radicals

For any real numbers a and b, $\sqrt{ab} = \sqrt{a}\sqrt{b}$.

This extends to any radicals (e.g. cube roots), but we will concentrate on square roots in this course.

Note the similarity to the distributive law for exponents, $(ab)^n = a^n b^n$.

In fact, radicals can be written as exponents. This is a topic covered in later courses.

Simplifying Radicals

Recall that a *perfect square* is an integer whose square root is also an integer.

The list of perfect squares are 1, 4, 9, 16, 25, etc.

If a is a perfect square, then $\sqrt{a}\sqrt{b}$ will become $k\sqrt{b}$, where k is an integer.

Therefore, to simplify $\sqrt{18}$ we must find two integers with a product of 18, such that one of them is a perfect square.

In this case, $18 = 9 \times 2$, so...

$$\begin{aligned}\sqrt{18} &= \sqrt{9 \times 2} \\ &= \sqrt{9}\sqrt{2} \\ &= 3\sqrt{2}\end{aligned}$$

Simplifying Radicals

Example

Simplify $\sqrt{40}$.

Express 40 as the product of two integers, where one is a perfect square.

In this case, $40 = 4 \times 10$.

$$\begin{aligned}\sqrt{40} &= \sqrt{4 \times 10} \\ &= \sqrt{4}\sqrt{10} \\ &= 2\sqrt{10}\end{aligned}$$

Simplifying Radicals

Example

Simplify $\sqrt{63}$.

Use $63 = 9 \times 7$, since 9 is a perfect square.

$$\begin{aligned}\sqrt{63} &= \sqrt{9 \times 7}\\ &= \sqrt{9}\sqrt{7}\\ &= 3\sqrt{7}\end{aligned}$$

Simplifying Radicals

Example

Simplify $\sqrt{15}$.

There are two ways to express 15 as the product of two positive integers: 1×15 and 3×5.

Since 1 is the only perfect square, $\sqrt{15}$ cannot be simplified any further.

Simplifying Radicals

Example

Simplify $\sqrt{128}$.

Sometimes it is easier to simplify large radicals in multiple stages.

Since 128 is divisible by 4 (a perfect square), start with $128 = 4 \times 32$.

$$\begin{aligned}\sqrt{128} &= \sqrt{4 \times 32}\\ &= \sqrt{4}\sqrt{32}\\ &= 2\sqrt{32}\end{aligned}$$

Simplifying Radicals

Since $32 = 16 \times 2$, and 16 is another perfect square, we can simplify the radical further.

$$\begin{aligned}\sqrt{128} &= 2\sqrt{32}\\ &= 2\sqrt{16 \times 2}\\ &= 2\sqrt{16}\sqrt{2}\\ &= (2 \times 4)\sqrt{2}\\ &= 8\sqrt{2}\end{aligned}$$

Note that we obtain the same answer if we go directly to $\sqrt{128} = \sqrt{64 \times 2} = 8\sqrt{2}$.

Simplifying Radicals

Example

Express $\sqrt{252}$ as a mixed radical.

Using $252 = 4 \times 63$ to start...

$$\begin{aligned}\sqrt{252} &= \sqrt{4 \times 63}\\ &= \sqrt{4}\sqrt{63}\\ &= 2\sqrt{63}\end{aligned}$$

...and since $63 = 9 \times 7$...

$$\begin{aligned}\sqrt{252} &= 2\sqrt{9 \times 7}\\ &= 2\sqrt{9}\sqrt{7}\\ &= (2 \times 3)\sqrt{7}\\ &= 6\sqrt{7}\end{aligned}$$

Questions?