MPM2D: Principles of Mathematics

Similar & Congruent Triangles

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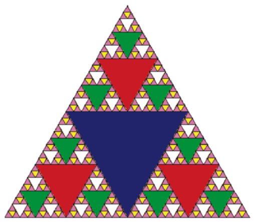
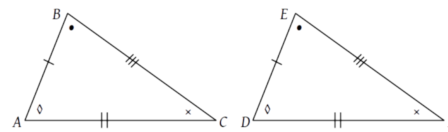


Congruent Triangles

Two triangles are *congruent* if they have the same shape, and the same size.

To meet both of these criteria, congruent triangles have equal corresponding angles and equal corresponding sides.

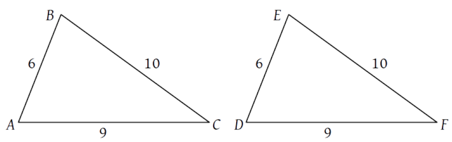


There are three distinct conditions that will result in congruent triangles.

Congruent Triangles

Side-Side-Side Congruency (SSS)

If $|AB| = |DE|$, $|AC| = |DF|$ and $|BC| = |EF|$, then $\Delta ABC \cong \Delta DEF$.

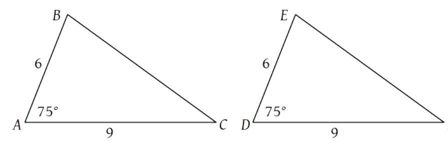


No matter how the sides are arranged, it will always be possible to find equal pairs of corresponding sides.

Congruent Triangles

Side-Angle-Side Congruency (SAS)

If $|AB| = |DE|$, $|AC| = |DF|$, and $\angle A = \angle D$, then $\Delta ABC \cong \Delta DEF$.

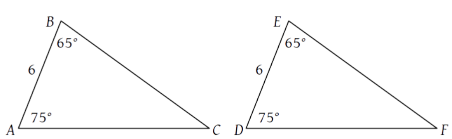


Two sides of a fixed length, containing a fixed angle, will always result in a third size of a specific length.

Congruent Triangles

Angle-Side-Angle Congruency (ASA)

If $\angle A = \angle D$, $\angle B = \angle E$ and $|AB| = |DE|$, then $\Delta ABC \cong \Delta DEF$.

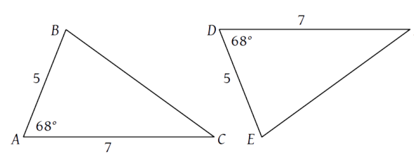


Like SAS, two fixed angles at each end of a side will result in two other sides of specific lengths.

Congruent Triangles

Example

State why $\Delta ABC \cong \Delta DEF$.

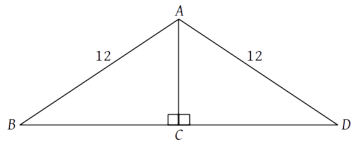


Since $|AB| = |DE| = 5$, $|AC| = |DF| = 7$ and $\angle A = \angle D$, $\Delta ABC \cong \Delta DEF$ due to SAS.

Congruent Triangles

Example

State why $\Delta ABC \cong \Delta ADC$.



Since $|AB| = |AD| = 12$, ΔABD is isosceles. This means that $\angle B = \angle D$.

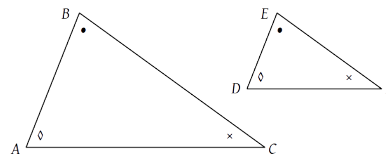
Since $\angle ACB = \angle ACD = 90°$, then $\angle BAC = \angle DAC$.

Therefore, $\Delta ABC \cong \Delta ADC$ due to ASA.

Similar Triangles

Two triangles are *similar* if they have the same overall shape.

Triangles with equal corresponding angles will have the same shape, but may be different sizes.

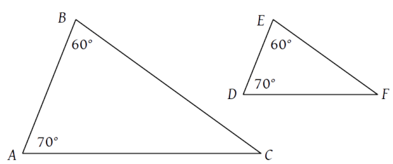


There are three distinct conditions that will result in similar triangles.

Similar Triangles

Angle-Angle Similarity (AA∼)

If $\angle A = \angle D$ and $\angle B = \angle E$, then $\Delta ABC \sim \Delta DEF$.

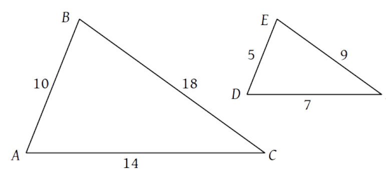


It is not necessary to specify that the remaining corresponding angles are equal, since it will always be true.

Similar Triangles

Side-Side-Side Similarity (SSS∼)

If $|AB| = k|DE|$, $|AC| = k|DF|$, and $|BC| = k|EF|$, then $\Delta ABC \sim \Delta DEF$.

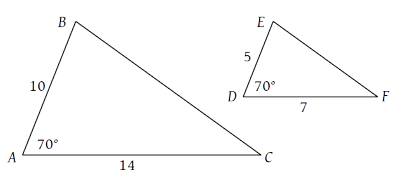


Since each side has been scaled by the same amount, the overall shape is preserved.

Similar Triangles

Side-Angle-Side Similarity (SAS∼)

If $|AB| = k|DE|$, $|AC| = k|DF|$, and $\angle A = \angle D$, then $\Delta ABC \sim \Delta DEF$.

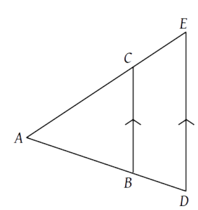


Since two sides containing the angle have been scaled by the same amount, the third side will also by scaled by the same amount as well.

Similar Triangles

Example

State why $\Delta ABC \sim \Delta ADE$.



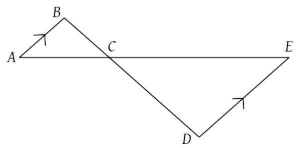
$\angle A$ is common to both triangles, and $\angle ACB = \angle AED$, since they are corresponding angles (F pattern).

Therefore, ΔABC sin ΔADE due to AA∼.

Similar Triangles

Example

State why $\triangle ABC \sim \triangle EDC$.



$\angle ACB = \angle ECD$, since they are opposite angles.

$\angle B = \angle F$, since they are alternate angles (Z pattern).

Therefore, $\triangle ABC \sin \triangle EDC$ due to AA$\sim$.

Questions?