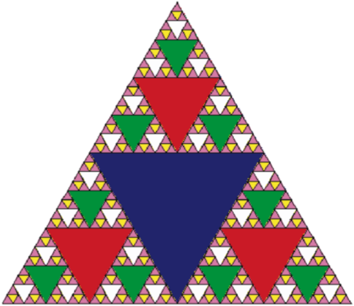
MPM2D: Principles of Mathematics

# Similar & Congruent Triangles

J. Garvin



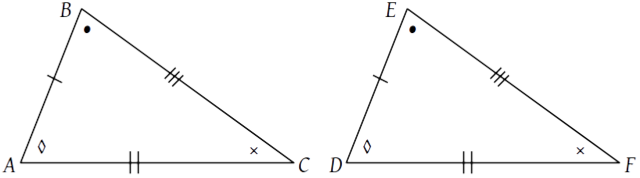
# Congruent Triangles

Two triangles are *congruent* if they have the same shape, and the same size.

# Congruent Triangles

Two triangles are *congruent* if they have the same shape, and the same size.

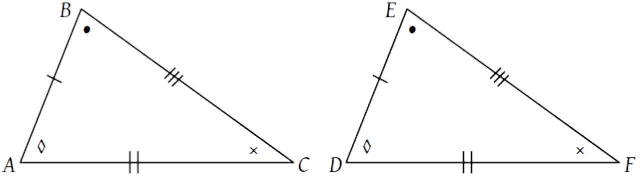
To meet both of these criteria, congruent triangles have equal corresponding angles and equal corresponding sides.



# Congruent Triangles

Two triangles are *congruent* if they have the same shape, and the same size.

To meet both of these criteria, congruent triangles have equal corresponding angles and equal corresponding sides.

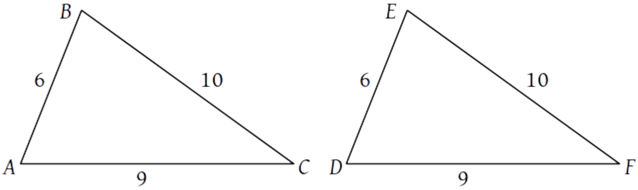



There are three distinct conditions that will result in congruent triangles.

# Congruent Triangles

## Side-Side-Side Congruency (SSS)

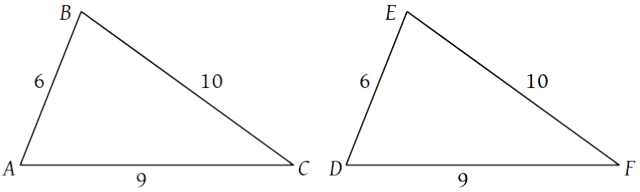
If $|AB| = |DE|$, $|AC| = |DF|$ and $|BC| = |EF|$, then $\Delta ABC \cong \Delta DEF$.



# Congruent Triangles

## Side-Side-Side Congruency (SSS)

If $|AB| = |DE|$, $|AC| = |DF|$ and $|BC| = |EF|$, then $\Delta ABC \cong \Delta DEF$.

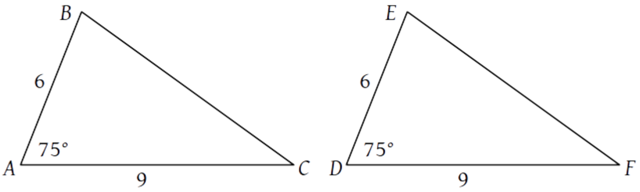



No matter how the sides are arranged, it will always be possible to find equal pairs of corresponding sides.

# Congruent Triangles

## Side-Angle-Side Congruency (SAS)

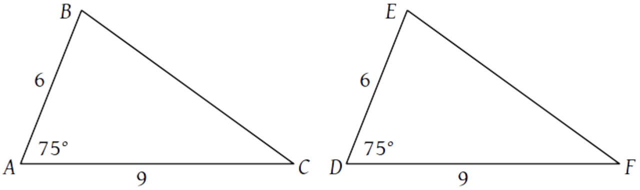
If $|AB| = |DE|$, $|AC| = |DF|$, and $\angle A = \angle D$, then $\Delta ABC \cong \Delta DEF$.



# Congruent Triangles

## Side-Angle-Side Congruency (SAS)

If $|AB| = |DE|$, $|AC| = |DF|$, and $\angle A = \angle D$, then $\Delta ABC \cong \Delta DEF$.

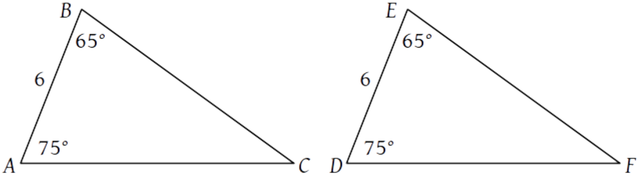



Two sides of a fixed length, containing a fixed angle, will always result in a third size of a specific length.

# Congruent Triangles

## Angle-Side-Angle Congruency (ASA)

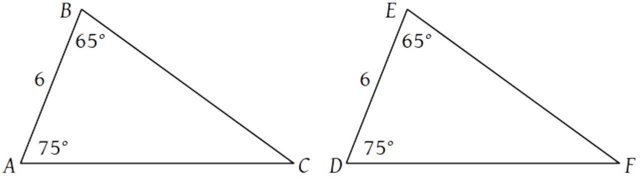
If $\angle A = \angle D$, $\angle B = \angle E$ and $|AB| = |DE|$, then $\Delta ABC \cong \Delta DEF$.



# Congruent Triangles

## Angle-Side-Angle Congruency (ASA)

If $\angle A = \angle D$, $\angle B = \angle E$ and $|AB| = |DE|$, then $\Delta ABC \cong \Delta DEF$.

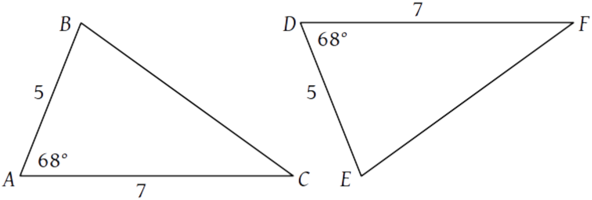



Like SAS, two fixed angles at each end of a side will result in two other sides of specific lengths.

# Congruent Triangles

## Example

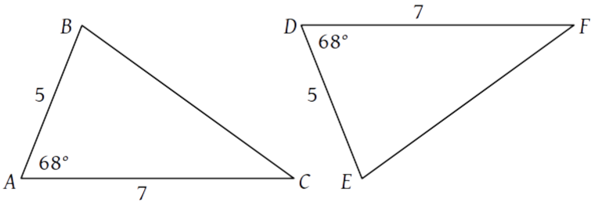
State why $\Delta ABC \cong \Delta DEF$.



# Congruent Triangles

## Example

State why $\Delta ABC \cong \Delta DEF$.

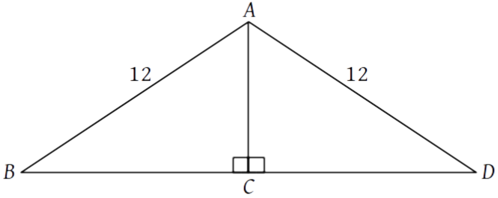



Since $|AB| = |DE| = 5$, $|AC| = |DF| = 7$ and $\angle A = \angle D$, $\Delta ABC \cong \Delta DEF$ due to SAS.

# Congruent Triangles

## Example

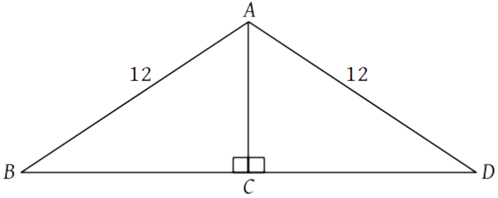
State why $\Delta ABC \cong \Delta ADC$.



# Congruent Triangles

## Example

State why $\Delta ABC \cong \Delta ADC$.

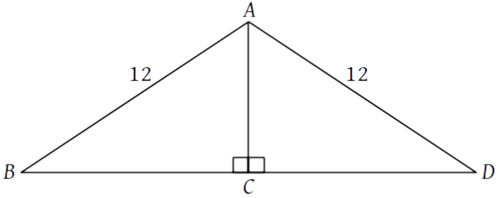



Since $|AB| = |AD| = 12$, $\Delta ABD$ is isosceles. This means that $\angle B = \angle D$.

# Congruent Triangles

## Example

State why $\Delta ABC \cong \Delta ADC$.



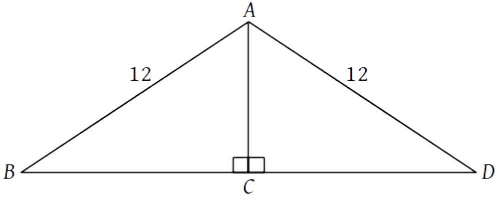

Since $|AB| = |AD| = 12$, $\Delta ABD$ is isosceles. This means that $\angle B = \angle D$.

Since $\angle ACB = \angle ACD = 90^\circ$, then $\angle BAC = \angle DAC$.

# Congruent Triangles

## Example

State why $\Delta ABC \cong \Delta ADC$.




Since $|AB| = |AD| = 12$, $\Delta ABD$ is isosceles. This means that $\angle B = \angle D$.

Since $\angle ACB = \angle ACD = 90^\circ$, then $\angle BAC = \angle DAC$.

$AC$ is common to both $\Delta ABC$ and $\Delta ADC$, so $\Delta ABC \cong \Delta ADC$ due to SAS.

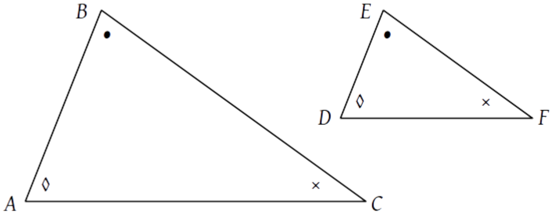
# Similar Triangles

Two triangles are *similar* if they have the same overall shape.

# Similar Triangles

Two triangles are *similar* if they have the same overall shape.

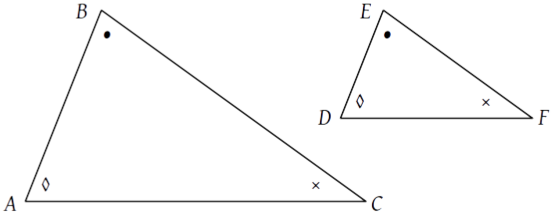
Triangles with equal corresponding angles will have the same shape, but may be different sizes.



# Similar Triangles

Two triangles are *similar* if they have the same overall shape.

Triangles with equal corresponding angles will have the same shape, but may be different sizes.

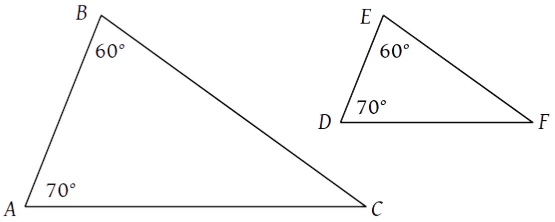



There are three distinct conditions that will result in similar triangles.

# Similar Triangles

## Angle-Angle Similarity (AA$\sim$)

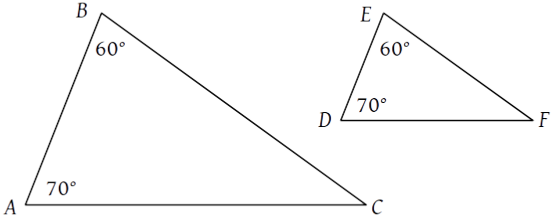
If $\angle A = \angle D$ and $\angle B = \angle E$, then $\Delta ABC \sim \Delta DEF$.



# Similar Triangles

## Angle-Angle Similarity (AA∼)

If $\angle A = \angle D$ and $\angle B = \angle E$, then $\Delta ABC \sim \Delta DEF$.

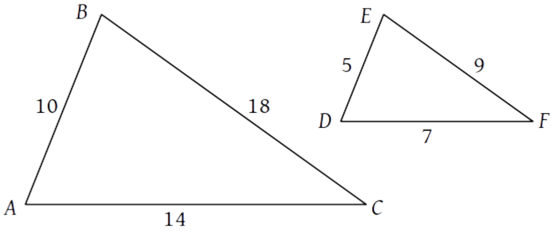



It is not necessary to specify that the remaining corresponding angles are equal, since it will always be true.

# Similar Triangles

## Side-Side-Side Similarity (SSS∼)

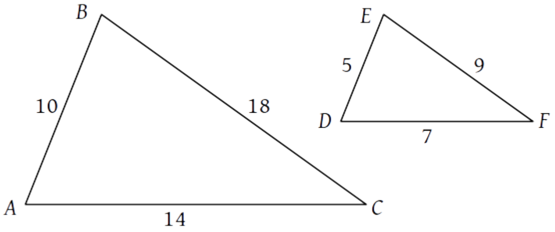
If $|AB| = k|DE|$, $|AC| = k|DF|$, and $|BC| = k|EF|$, then $\Delta ABC \sim \Delta DEF$.



# Similar Triangles

## Side-Side-Side Similarity (SSS∼)

If $|AB| = k|DE|$, $|AC| = k|DF|$, and $|BC| = k|EF|$, then $\Delta ABC \sim \Delta DEF$.

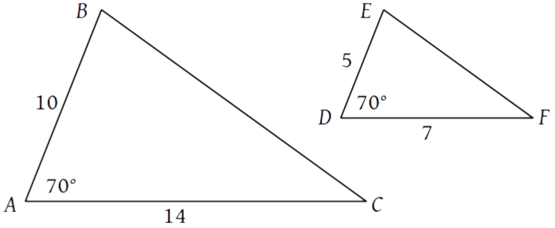



Since each side has been scaled by the same amount, the overall shape is preserved.

# Similar Triangles

## Side-Angle-Side Similarity (SAS$\sim$)

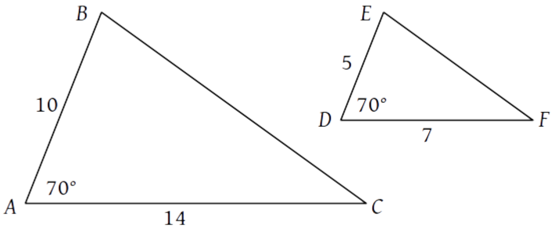
If $|AB| = k|DE|$, $|AC| = k|DF|$, and $\angle A = \angle D$, then $\Delta ABC \sim \Delta DEF$.



# Similar Triangles

## Side-Angle-Side Similarity (SAS$\sim$)

If $|AB| = k|DE|$, $|AC| = k|DF|$, and $\angle A = \angle D$, then $\Delta ABC \sim \Delta DEF$.

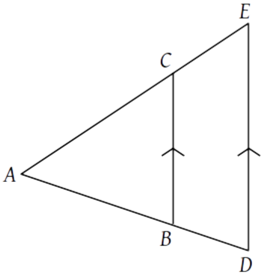



Since two sides containing the angle have been scaled by the same amount, the third side will also by scaled by the same amount as well.

# Similar Triangles

## Example

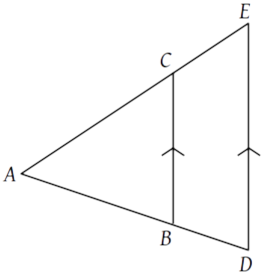
State why $\Delta ABC \sim \Delta ADE$.



# Similar Triangles

## Example

State why $\Delta ABC \sim \Delta ADE$.

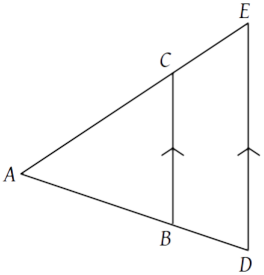



$\angle A$ is common to both triangles, and $\angle ACB = \angle AED$, since they are corresponding angles (F pattern).

# Similar Triangles

## Example

State why $\Delta ABC \sim \Delta ADE$.



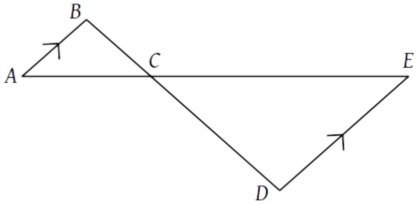

$\angle A$ is common to both triangles, and $\angle ACB = \angle AED$, since they are corresponding angles (F pattern).

Therefore, $\Delta ABC$ sin $\Delta ADE$ due to AA$\sim$.

# Similar Triangles

## Example

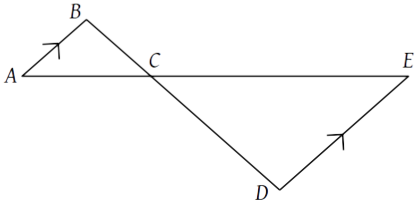
State why $\Delta ABC \sim \Delta EDC$.



# Similar Triangles

## Example

State why $\Delta ABC \sim \Delta EDC$.

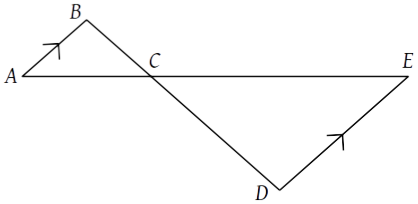



$\angle ACB = \angle ECD$, since they are opposite angles.

# Similar Triangles

## Example

State why $\Delta ABC \sim \Delta EDC$.



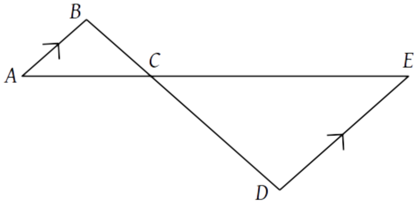

$\angle ACB = \angle ECD$, since they are opposite angles.

$\angle B = \angle F$, since they are alternate angles (Z pattern).

# Similar Triangles

## Example

State why $\Delta ABC \sim \Delta EDC$.




$\angle ACB = \angle ECD$, since they are opposite angles.

$\angle B = \angle F$, since they are alternate angles (Z pattern).

Therefore, $\Delta ABC \sin \Delta EDC$ due to AA$\sim$.

# Questions?