MPM2D: Principles of Mathematics

Ratios and Proportions

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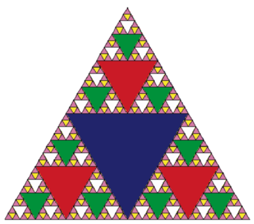

Solving Proportions

A *ratio* is a comparison between two quantities.

For example, the instructions on a can of orange juice concentrate may read "add 1 can of concentrate to 3 cans of water."

The ratio of concentrate to water can be expressed using a colon, $1 : 3$, or as a fraction, $\frac{1}{3}$.

Note that this fraction does *not* mean that the orange juice is one-third concentrate.

The orange juice is actually one-fourth concentrate, since there were four cans added: 1 of concentrate, and 3 of water.

While this often makes it more convenient to express ratios using a colon for common measurements, it is usually more advantageous for us to use fractions for our calculations.

Solving Proportions

A *proportion* is a statement that equates two or more ratios.

For example, $\frac{2}{9} = \frac{6}{27}$ is a proportion, since the ratio on the right can be reduced to the ratio on the left.

In general, two ratios are equal if there is some scale factor k, such that $\frac{k}{k} \cdot \frac{a}{b} = \frac{c}{d}$.

In this case, since $\frac{3}{3} \cdot \frac{2}{9} = \frac{6}{27}$, $k = 3$.

Many proportions can be solved by identifying the scale factor, then working out any missing values.

Solving Proportions

Example

Solve $\dfrac{x}{15} = \dfrac{2}{5}$.

Since $5 \times 3 = 15$, $k = 3$.

Therefore, $x = 2 \times k = 2 \times 3 = 6$.

Example

Solve $\dfrac{x}{12} = \dfrac{5}{8}$.

Since $8 \times \frac{3}{2} = 12$, $k = \frac{3}{2}$.

Therefore, $x = 5 \times k = 5 \times \frac{3}{2} = \frac{15}{2}$.

Solving Proportions

A useful technique, known as *cross-multiplication*, can be used to eliminate two different denominators simultaneously.

$$\frac{a}{b} = \frac{c}{d}$$
$$\frac{a}{b} \cdot b = \frac{c}{d} \cdot b$$
$$a = \frac{bc}{d}$$
$$a \cdot d = \frac{bc}{d} \cdot d$$
$$a \cdot d = b \cdot c$$

Cross-Multiplication

If $\dfrac{a}{b} = \dfrac{c}{d}$, then $a \cdot d = b \cdot c$.

Solving Proportions

Example

Solve $\dfrac{x}{12} = \dfrac{5}{8}$ using cross-multiplication.

$$\frac{x}{12} = \frac{5}{8}$$
$$8x = 12 \cdot 5$$
$$8x = 60$$
$$x = \tfrac{60}{8}$$
$$x = \tfrac{15}{2}$$

Solving Proportions

Example

Solve $\frac{x}{4} = \frac{5}{7}$.

$$\begin{aligned} \frac{x}{4} &= \frac{5}{7} \\ 7x &= 4 \cdot 5 \\ 7x &= 20 \\ x &= \tfrac{20}{7} \end{aligned}$$

Solving Proportions

Example

Solve $\frac{x+1}{8} = \frac{4}{3}$.

Be sure to use the distributive law when cross-multiplying.

$$\begin{aligned} \frac{x+1}{8} &= \frac{4}{3} \\ 3(x+1) &= 8 \cdot 4 \\ 3x+3 &= 32 \\ 3x &= 29 \\ x &= \tfrac{29}{3} \end{aligned}$$

Word Problems Involving Proportions

Many types of word problems involve proportions.

Information will usually be given about one ratio, while the remaining ratio will require solving.

Proportion problems usually have the format "if a is to b, then c is to..."

There may be more than one way to solve the problem. As long as the ratios are valid, solving a proportion using the previous techniques should work.

Word Problems Involving Proportions

Example

If it costs $48.00 to buy 5 shirts, how much should it cost to buy a dozen shirts?

Set up a proportion comparing the costs of the shirts, c, to the number of shirts.

$$\begin{aligned} \frac{c}{12} &= \frac{48}{5} \\ 5c &= 12 \cdot 48 \\ 5c &= 576 \\ c &= 115.2 \end{aligned}$$

A dozen shirts should cost $115.20.

Word Problems Involving Proportions

Example

A pallet of bricks has a mass of 10 000 kg. After 1 680 bricks have been used, the pallet has a mass of 5 800 kg. How many bricks were on the pallet?

Set up a proportion comparing the number of bricks, b, to their masses.

$$\begin{aligned} \frac{b}{10\,000} &= \frac{b-1\,680}{5\,800} \\ 5\,800b &= 10\,000(b-1\,680) \\ 5\,800b &= 10\,000b - 16\,800\,000 \\ 4\,200b &= 16\,800\,000 \\ b &= 4\,000 \end{aligned}$$

A total of 4 000 bricks were on the pallet.

Questions?