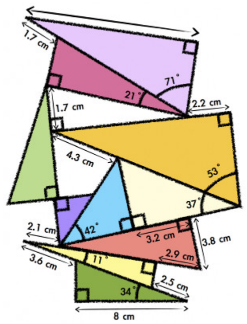
MPM2D: Principles of Mathematics

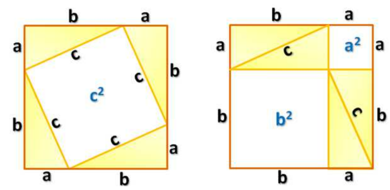
The Pythagorean Theorem

J. Garvin



Pythagorean Theorem

Consider four congruent triangles with arms a and b and hypotenuses c. They can be arranged in many ways, including the two below.

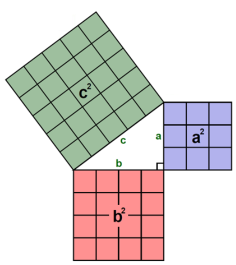


The white area on the left, c^2, is the same as the sum of the white areas on the right, $a^2 + b^2$.

Pythagorean Theorem

Pythagorean Theorem

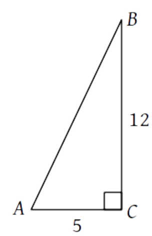
If a and b are the arms in right triangle ABC, and c is the hypotenuse, then $a^2 + b^2 = c^2$.



Pythagorean Theorem

Example

Determine $|AB|$ in the triangle below.

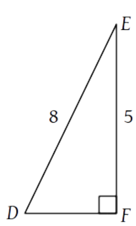


$$\begin{aligned} |AC|^2 + |BC|^2 &= |AB|^2 \\ 5^2 + 12^2 &= |AB|^2 \\ 169 &= |AB|^2 \\ \sqrt{169} &= |AB| \\ 13 &= |AB| \end{aligned}$$

Pythagorean Theorem

Example

Determine $|DF|$ in the triangle below.

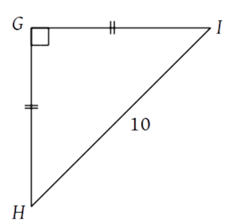


$$\begin{aligned} |DF|^2 + |EF|^2 &= |DE|^2 \\ |DF|^2 + 5^2 &= 8^2 \\ |DF|^2 &= 64 - 25 \\ |DF| &= \sqrt{39} \\ |DF| &\approx 6.245 \end{aligned}$$

Pythagorean Theorem

Example

Determine $|GH|$ in the triangle below.

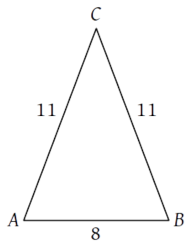


$$\begin{aligned} |GH|^2 + |GI|^2 &= |HI|^2 \\ 2|GH|^2 &= 10^2 \\ 2|GH|^2 &= 100 \\ |GH|^2 &= 50 \\ |GH| &= \sqrt{50} \\ |GH| &\approx 7.071 \end{aligned}$$

Pythagorean Theorem

Example

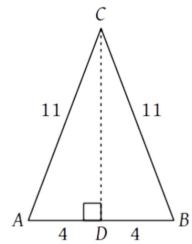
Determine the area of ΔABC below.



To determine the area of ΔABC, we first need to determine its height.

Since the triangle is isosceles, the height will *bisect* AB at $90°$.

Pythagorean Theorem



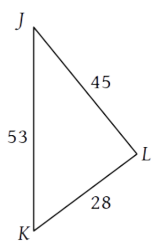
$$\begin{aligned} |AD|^2 + |CD|^2 &= |AC|^2 \\ 4^2 + |CD|^2 &= 11^2 \\ |CD|^2 &= 105 \\ |CD| &= \sqrt{105} \\ |CD| &\approx 10.247 \end{aligned}$$

Using the formula $A = \frac{1}{2}bh$, the area of ΔABC is $A = \frac{1}{2} \times 8 \times \sqrt{105} \approx 40.988$ square units.

Pythagorean Theorem

Example

Verify that ΔJKL contains a right angle.



If ΔJKL contains a right angle, the Pythagorean Theorem will hold true.

The sum of the squares of the arms is $28^2 + 45^2 = 2\,809$.

The square of the hypotenuse is $53^2 = 2\,809$.

Since we obtain the same value, $\angle L = 90°$, as it is across from the hypotenuse.

Questions?