MPM2D: Principles of Mathematics

Classifying & Verifying Properties of Triangles

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Classifying Triangles

Triangles are generally classified in two ways.

1. Based on the magnitudes of its sides:
   - Equilateral (three equal sides/angles)
   - Isosceles (two equal sides)
   - Scalene (all side lengths are distinct)
2. Based on the magnitudes of its angles:
   - Right (one 90° angle)
   - Acute (all angles less than 90°)
   - Obtuse (one angle greater than 90°)

It is possible to classify triangles using the formulae for slopes, side lengths and midpoints.

Classifying Triangles

Example

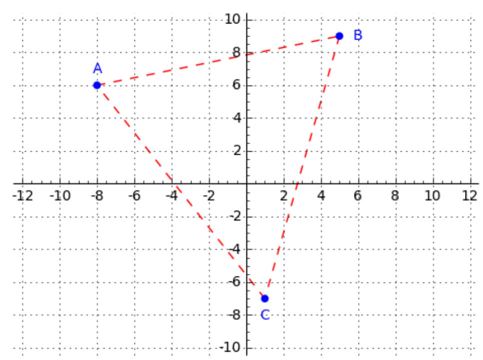
Classify the triangle with vertices $A(-8,6)$, $B(5,9)$ and $C(1,-7)$ as equilateral, isosceles or scalene.

Calculate $|AB|$, $|BC|$, and $|AC|$ to identify any equal side lengths.

$$|AB| = \sqrt{(5-(-8))^2+(9-6)^2} = \sqrt{178}$$
$$|BC| = \sqrt{(1-5)^2+(-7-9)^2} = 4\sqrt{17}$$
$$|AC| = \sqrt{(1-(-8))^2+(-7-6)^2} = 5\sqrt{10}$$

Since all side lengths are different, ΔABC is scalene.

Classifying Triangles



Classifying Triangles

Example

Determine whether the triangle with vertices $P(-4,4)$, $Q(2,8)$ and $R(4,-8)$ contains a right angle.

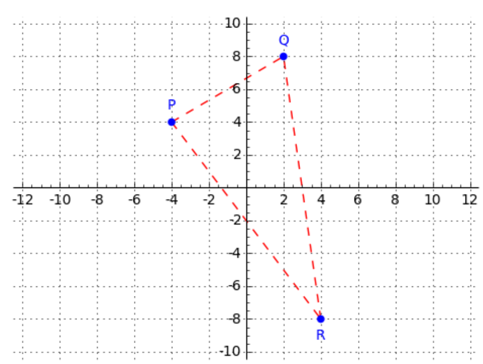
A triangle with a right angle will have two sides that are perpendicular.

These sides will have negative reciprocal slopes.

$$m_{PQ} = \frac{8-4}{2-(-4)} = \frac{2}{3} \qquad m_{QR} = \frac{-8-8}{4-2} = -8 \qquad m_{PR} = \frac{-8-4}{4-(-4)} = -\frac{3}{2}$$

Since $PQ \perp PR$, $\angle P = 90°$.

Classifying Triangles



Classifying Triangles

Example

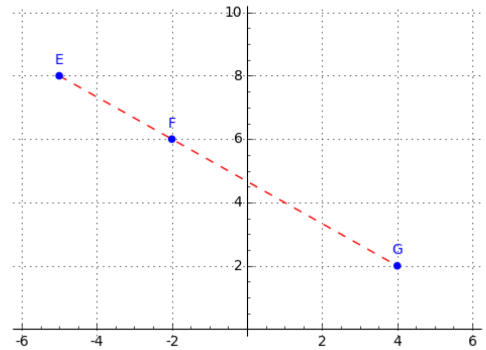
Show that it is not possible for the points $E(-5, 8)$, $F(-2, 6)$ and $G(4, 2)$ to form a triangle.

Since any three *non-collinear* points will form a triangle, check the slopes of any two line segments to see if the three points are indeed collinear.

$$m_{EF} = \frac{6-8}{-2-(-5)} = -\frac{2}{3} \qquad m_{FG} = \frac{2-6}{4-(-2)} = -\frac{2}{3}$$

Since the slopes are the same, the three points are collinear, and cannot form a triangle.

Classifying Triangles

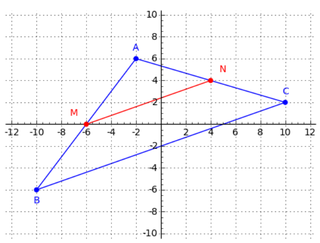


Properties of Triangles

A useful property is the *Triangle Midpoint Theorem*.

Triangle Midpoint Theorem

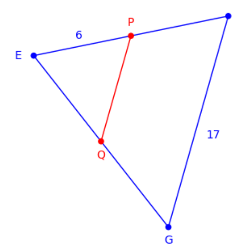
If M and N are the midpoints of AB and AC in ΔABC, then $|BC| = 2|MN|$ and $BC \parallel MN$.



Properties of Triangles

Example

In the diagram below, P and Q are the midpoints of EF and EG. Determine $|EF|$ and $|PQ|$.



Properties of Triangles

Since P is the midpoint of EF, $|EF| = 2|EP|$.

$$|EF| = 2 \times 6 = 12$$

Since PQ connects the midpoints of EF and EG, $|PQ| = \frac{1}{2}|FG|$.

$$|EF| = \tfrac{1}{2}(17) = 8.5$$

This theorem will be revisited when we explore *similar triangles* later in the course.

Questions?