MPM2D: Principles of Mathematics

Properties of Similar Triangles

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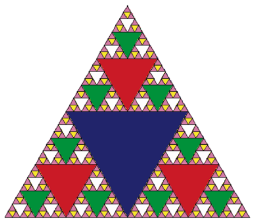
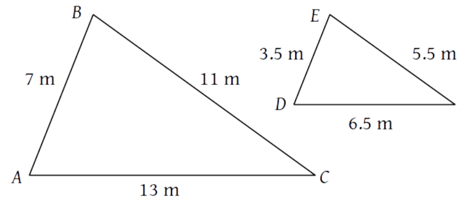


Solving Proportions

Recap

Explain why $\Delta ABC \sim \Delta DEF$.



Each side in ΔABC is twice as long as that in ΔDEF. Therefore, $\Delta ABC \sim \Delta DEF$ by SSS$\sim$.

Solving Proportions

Since the ratios of any two corresponding sides of two similar triangles are equal, we can use these ratios to solve for the values of unknown sides.

Consider the case where $\Delta ABC \sim \Delta DEF$.

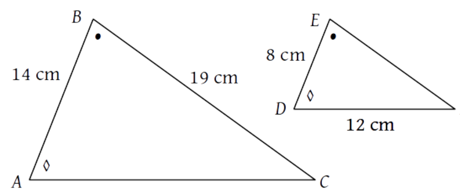
If we know the ratio $\frac{|AB|}{|DE|}$, and we know one of $|AC|$ or $|DF|$, then we can solve for the other value using the proportion $\frac{|AB|}{|DE|} = \frac{|AC|}{|DF|}$.

This can be done by inspection, or by using cross-multiplication.

Solving Similar Triangles

Example

$\Delta ABC \sim \Delta DEF$. Determine $|AC|$ and $|EF|$.



Since we know the values of the corresponding sides AB and DE, we can use their ratio to create proportions involving the unknown sides.

Solving Similar Triangles

AC corresponds to DF, whose value is known.

$$\frac{|AB|}{|DE|} = \frac{|AC|}{|DF|}$$
$$\frac{14}{8} = \frac{|AC|}{12}$$
$$168 = 8|AC|$$
$$|AC| = 21$$

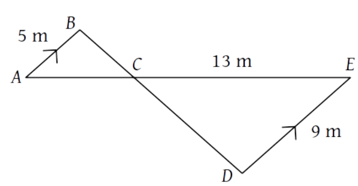
EF corresponds to BC, whose value is known.

$$\frac{|AB|}{|DE|} = \frac{|BC|}{|EF|}$$
$$\frac{14}{8} = \frac{19}{|EF|}$$
$$14|EF| = 152$$
$$|EF| = \tfrac{76}{7}$$

Solving Similar Triangles

Example

Determine $|AE|$.



Since $|AE| = |AC| + |CE|$, we can find $|AC|$ and add it to $|CE|$, which is known.

Solving Similar Triangles

AB corresponds with DE, while AC corresponds with CE.

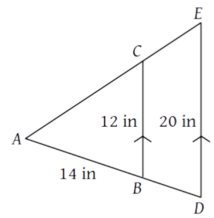
$$\frac{|AB|}{|DE|} = \frac{|AC|}{|CE|}$$
$$\frac{5}{9} = \frac{|AC|}{13}$$
$$65 = 9|AC|$$
$$|AC| = \tfrac{65}{9}$$

Therefore, $|AE| = 13 + \frac{65}{9} = \frac{182}{9}$ m.

Solving Similar Triangles

Example

Determine $|BD|$.



Since BD is part of a trapezoid rather than a triangle, we cannot use it directly in a proportion.

Solving Similar Triangles

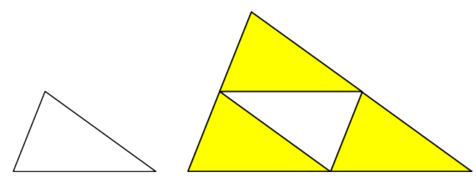
Since $|BD| = |AD| - |AB|$, we can first find $|AD|$ and subtract $|AB|$, which is known.

$$\frac{|AD|}{|AB|} = \frac{|DE|}{|BC|}$$
$$\frac{|AD|}{14} = \frac{20}{12}$$
$$\frac{|AD|}{14} = \frac{5}{3}$$
$$3|AD| = 70$$
$$|AD| = \tfrac{70}{3}$$

Therefore, $|BD| = \frac{70}{3} - 14 = \frac{28}{3}$ in.

Areas of Similar Triangles

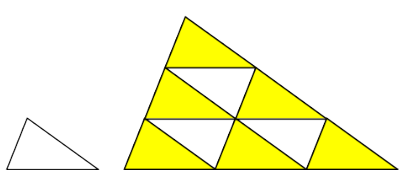
What happens to the area of a triangle when its dimensions are doubled?



When the dimensions are doubled, the area is quadrupled.

Areas of Similar Triangles

What happens to the area of a triangle when its dimensions are tripled?



When the dimensions are tripled, the area increases by a factor of nine.

Areas of Similar Triangles

In general, a triangle whose dimensions are enlarged by a factor of k will have an area k^2 times greater.

Areas of Similar Triangles

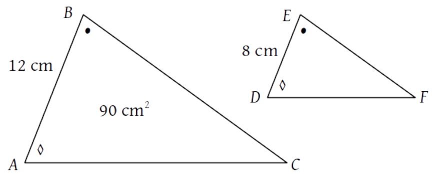
If $\Delta ABC \sim \Delta DEF$, and if $|AB| = k|DE|$, then $\text{Area}_{ABC} = k^2 \cdot \text{Area}_{DEF}$.

Any ratio of corresponding sides can be used, so choose the one that is easiest to work with.

Areas of Similar Triangles

Example

$\Delta ABC \sim \Delta DEF$. Determine Area_{DEF}.



Areas of Similar Triangles

Determine the scale factor, k, of ΔDEF.

$$12k = 8$$
$$k = \frac{2}{3}$$

The area of ΔDEF will be $k^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$ the size of ΔABC.

Therefore, the area of ΔDEF is $90 \times \frac{4}{9} = 40$ cm^2.

Questions?