MPM2D: Principles of Mathematics

Distributive Law

Products of Two Binomials

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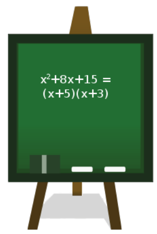

Evaluating Expressions w/ the Distributive Law

Consider the expression 4×7.

Multiplying, $4 \times 7 = 28$.

Now consider the expression $4(2+5)$.

Evaluating inside of the brackets first gives $4(2+5) = 4 \times 7 = 28$ as before.

It is also possible (but not recommended) to use the Distributive Law to evaluate the expression.

$$\begin{aligned} 4(2+5) &= 4 \cdot 2 + 4 \cdot 5 \\ &= 8 + 20 \\ &= 28 \end{aligned}$$

Evaluating Expressions w/ the Distributive Law

Now consider the expression $(1+3)(2+5)$.

Evaluating inside of the brackets, as we should, gives $(1+3)(2+5) = 4 \cdot 7 = 28$ as expected.

How could the Distributive Law be used here?

To evaluate the expression using the Distributive Law, each term in the first pair of brackets is multiplied by each term in the second pair.

$$\begin{aligned} (1+3)(2+5) &= 1 \cdot 2 + 1 \cdot 5 + 3 \cdot 2 + 3 \cdot 5 \\ &= 2 + 5 + 6 + 15 \\ &= 28 \end{aligned}$$

Evaluating Expressions w/ the Distributive Law

Example

Use the Distributive Law to evaluate $(2+6)(4+1)$, and verify the solution by evaluating within the brackets first.

Multiply each term in the first pair of brackets by each term in the second.

$$\begin{aligned} (2+6)(4+1) &= 2 \cdot 4 + 2 \cdot 1 + 6 \cdot 4 + 6 \cdot 1 \\ &= 8 + 2 + 24 + 6 \\ &= 40 \end{aligned}$$

Evaluating inside of the brackets first, $(2+6)(4+1) = 8 \cdot 5 = 40$.

Products of Two Binomials

Clearly, this is more work than necessary for evaluating an expression, so when is this useful?

Consider the expression $(x+1)(x+3)$.

Within the first pair of brackets, x and 1 are unlike terms, so they cannot be simplified.

The same is true of x and 3 in the second pair of brackets.

Both $x+1$ and $x+3$ are *binomials* – they are *polynomials* that contain 2 terms.

The Distributive Law allows us to rewrite the product of two binomials as a single expression instead.

Products of Two Binomials

Using the Distributive Law as before, multiply each term in the first pair of brackets by each term in the second.

$$\begin{aligned} (x+1)(x+3) &= x \cdot x + 3 \cdot x + 1 \cdot x + 1 \cdot 3 \\ &= x^2 + 3x + x + 3 \\ &= x^2 + 4x + 3 \end{aligned}$$

The new expression is a *quadratic* expression, and has the general form $ax^2 + bx + c$ for some real values a, b amd c.

In this case, the quadratic expression we obtained is a *trinomial*, since it contains three terms.

Products of Two Binomials

Example

Expand and simplify $(x+2)(x+5)$.

$$\begin{aligned}(x+2)(x+5) &= x \cdot x + 5 \cdot x + 2 \cdot x + 2 \cdot 5 \\ &= x^2 + 5x + 2x + 10 \\ &= x^2 + 7x + 10\end{aligned}$$

Note that all signs are positive in the final expression, since both the sum and the product of two positive numbers are positive.

Products of Two Binomials

Example

Expand and simplify $(x-3)(x-6)$.

$$\begin{aligned}(x-3)(x-6) &= x^2 - 6x - 3x + 18 \\ &= x^2 - 9x + 18\end{aligned}$$

In this example, the sum of two negative numbers is negative, while the product of two negative numbers is positive.

Products of Two Binomials

Example

Expand and simplify $(x+7)(x-5)$.

$$\begin{aligned}(x+7)(x-5) &= x^2 - 5x + 7x - 35 \\ &= x^2 + 2x - 35\end{aligned}$$

In this example, the product of a positive number and a negative number is negative.

The sum of the two numbers (one positive, one negative) is positive, because the positive number has a greater magnitude.

Products of Two Binomials

Example

Expand and simplify $(x-3)^2$.

Using the definition of exponentiation, $a^2 = a \cdot a$.

Therefore, $(x-3)^2 = (x-3)(x-3)$.

$$\begin{aligned}(x-3)(x-3) &= x^2 - 3x - 3x + 9 \\ &= x^2 - 6x + 9\end{aligned}$$

Note that $(x-3)^2 \neq x^2 - 9$, since the middle term is absent. This is a common error.

Products of Two Binomials

Example

Expand and simplify $(2x+1)(3x-4)$.

Even with coefficients, follow the same procedure as before.

$$\begin{aligned}(2x+1)(3x-4) &= 2x \cdot 3x + 2x \cdot (-4) + 3x \cdot 1 + 1 \cdot (-4) \\ &= 6x^2 - 8x + 3x - 4 \\ &= 6x^2 - 5x - 4\end{aligned}$$

Products of Two Binomials

Example

Expand and simplify $(3x-2)(5x-1)$.

$$\begin{aligned}(3x-2)(5x-1) &= 15x^2 - 3x - 10x + 2 \\ &= 15x^2 - 13x + 2\end{aligned}$$

Products of Two Binomials

Example

Expand and simplify $(2a+b)(a-3b)$.

Even with multiple variables, follow the same procedure.

$$\begin{aligned}(2a+b)(a-3b) &= 2a\cdot a + 2a\cdot(-3b) + b\cdot a + b\cdot(-3b)\\ &= 2a^2 - 6ab + ab - 3b^2\\ &= 2a^2 - 5ab - 3b^2\end{aligned}$$

Note that the middle term of the trinomial involves both a and b, whereas the other terms only contain one variable.

Products of Two Binomials

Example

Expand and simplify $(4p+5q)(2p+3q)$.

$$\begin{aligned}(4p+5q)(2p+3q) &= 8p^2 + 12pq + 10pq + 15q^2\\ &= 8p^2 + 22pq + 15q^2\end{aligned}$$

Questions?