MPM2D: Principles of Mathematics

Primary Trigonometric Ratios

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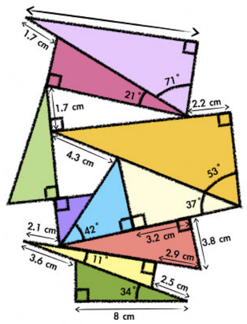
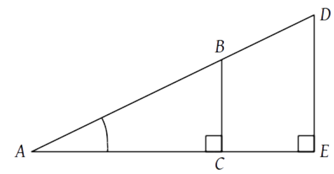


Similar Triangles

In the diagram below, $\Delta ABC \sim \Delta ADE$ since $\angle A$ is common to both triangles, and $\angle ACB = \angle AED$.



This means that any ratio of two sides in ΔABC is equal to the ratio of corresponding sides in ΔADE.

Similar Triangles

By varying the measure of $\angle A$, the ratio of two sides in ΔABC will change, but will remain equal to the ratio of corresponding sides in ΔADE.

Therefore, a specific measure of $\angle A$ can be associated with a specific ratio of two sides in a right triangle.

Is the ratio of two sides associated with a given angle unique?

Consider the ratio of the opposite side to the hypotenuse.

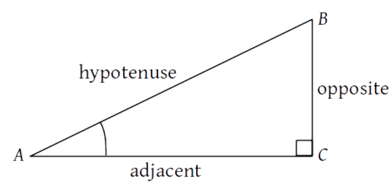
If $\angle A$ increases, the length of the opposite side also increases. Thus, the ratio will increase.

If $\angle A$ decreases, the length of the opposite side also decreases. Thus, the ratio will decrease.

In both scenarios, the ratio changes with the measure of $\angle A$. Therefore, the ratio associated with a specific angle is unique.

Primary Trigonometric Ratios

In the right triangle ΔABC below, the three sides have been labelled based on their position relative to $\angle A$.



The opposite and adjacent sides are reversed relative to $\angle B$, but the hypotenuse is *always* across from the right angle.

Primary Trigonometric Ratios

There are six possible ratios of sides that can be made from the three sides.

The three *primary trigonometric ratios* are *sine*, *cosine* and *tangent*.

Primary Trigonometric Ratios

Let ΔABC be a right triangle with $\angle A \neq 90°$. Then, the three primary trigonometric ratios for $\angle A$ are:

Sine: $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$

Cosine: $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$

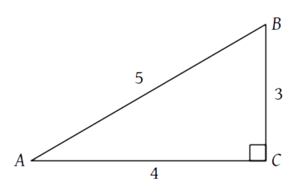
Tangent: $\tan A = \frac{\text{opposite}}{\text{adjacent}}$

The phrase *SOH-CAH-TOA* is a mnemonic for these ratios.

Primary Trigonometric Ratios

Example

State the three primary trigonometric ratios for $\angle A$ in ΔABC.



$$\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{3}{5}$$

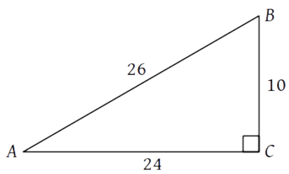
$$\cos A = \frac{\text{adj}}{\text{hyp}} = \frac{4}{5}$$

$$\tan A = \frac{\text{opp}}{\text{adj}} = \frac{3}{4}$$

Primary Trigonometric Ratios

Example

State the three primary trigonometric ratios for $\angle A$ in $\triangle ABC$. Express all ratios in simplest form.



$$\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{5}{13}$$

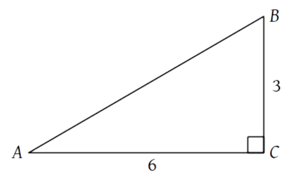
$$\cos A = \frac{\text{adj}}{\text{hyp}} = \frac{12}{13}$$

$$\tan A = \frac{\text{opp}}{\text{adj}} = \frac{5}{12}$$

Primary Trigonometric Ratios

Example

State the three primary trigonometric ratios for $\angle A$ in $\triangle ABC$.



Use the Pythagorean Theorem to determine the length of the hypotenuse, h.

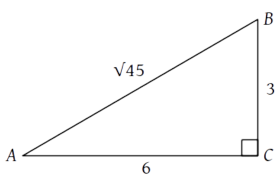
$$h^2 = 6^2 + 3^2$$

$$h^2 = 45$$

$$h = \sqrt{45}$$

Primary Trigonometric Ratios

This gives us the following right triangle.



$$\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{3}{\sqrt{45}}$$

$$\cos A = \frac{\text{adj}}{\text{hyp}} = \frac{6}{\sqrt{45}}$$

$$\tan A = \frac{\text{opp}}{\text{adj}} = \frac{3}{6} = \frac{1}{2}$$

Questions?