MPM2D: Principles of Mathematics

Partial Factoring

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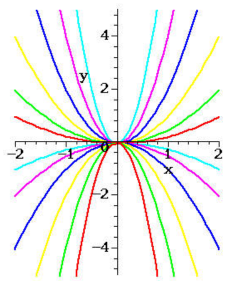


Quadratic Relations

Recall

Determine the location of the vertex for $y = 3x^2 + 18x$ by factoring.

Common factoring $3x$ from each term, we obtain $y = 3x(x + 6)$.

This relation has x-intercepts at $(0, 0)$ and $(-6, 0)$.

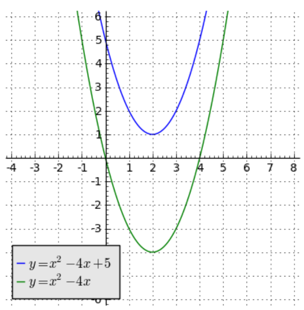
The vertex is midway between the x-intercepts, with an x-coordinate of $\frac{0-6}{2} = -3$.

The y-coordinate of the vertex is $3(-3)(-3 + 6) = -27$.

Therefore, the vertex is at $(-3, -27)$.

Partial Factoring

Consider the graphs of $y = x^2 - 4x + 5$ and $y = x^2 - 4x$ below.



Partial Factoring

The graph of $y = x^2 - 4x + 5$ has the same shape and orientation as the graph of $y = x^2 - 4x$, but it has been vertically shifted upward.

This means that while the vertices of the two parabolas have different y-coordinates, they have the same x-coordinates.

The relation $y = x^2 - 4x + 5$ cannot be factored, but $y = x^2 - 4x$ factors as $y = x(x - 4)$.

The x-coordinate of the vertex of $y = x^2 - 4x$ is $\frac{0+4}{2} = 2$.

Since the x-coordinates are the same for both vertices, the x-coordinate of the vertex of $y = x^2 - 4x + 5$ is also 2.

This technique of factoring a related quadratic is called *partial factoring*. It can help find the location of the vertex of a non-factorable quadratic relation. It *cannot* be used to find the x-intercepts – they may not exist!

Partial Factoring

Example

Determine the location of the vertex for $y = x^2 - 6x + 10$.

The simple trinomial cannot be factored, but the relation $y = x^2 - 6x$ can:

$$y = x(x - 6)$$

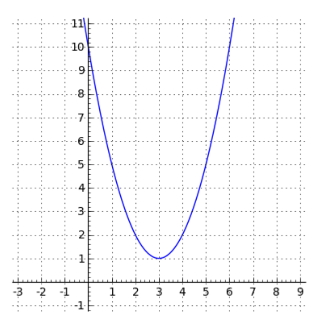
This relation has x-intercepts at $(0, 0)$ and $(6, 0)$, so the x-coordinate of the vertex is $\frac{0+6}{2} = 3$.

The y-coordinate of the vertex is $3^2 - 6(3) + 10 = 1$.

Therefore, the vertex is at $(3, 1)$.

Partial Factoring

A graph of $y = x^2 - 6x + 10$ is below. Note that it has no x-intercepts.



Partial Factoring

Example

Determine the location of the vertex for $y = 2x^2 + 16x + 27$.

The complex trinomial cannot be common factored, nor can it be factored using decomposition.

Using the relation $y = 2x^2 + 16x$, factor as:

$$y = 2x(x + 8)$$

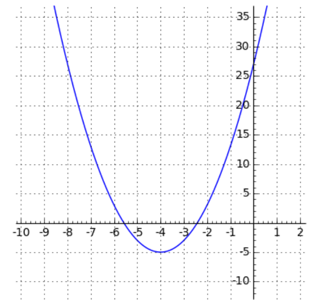
This relation has x-intercepts at $(0, 0)$ and $(-8, 0)$, so the x-coordinate of the vertex is $\frac{0-8}{2} = -4$.

The y-coordinate of the vertex is $2(-4)^2 + 16(-4) + 27 = -5$.

Therefore, the vertex is at $(-4, -5)$.

Partial Factoring

A graph of $y = 2x^2 + 16x + 27$ is below. Note that its x-intercepts are not integral.



Partial Factoring

Example

Sketch a graph of $y = -2x^2 + 20x - 37$.

Again, the complex trinomial cannot be factored.

Using the relation $y = -2x^2 + 20x$, factor as:

$$y = -2x(x - 10)$$

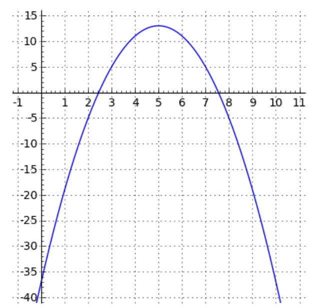
This relation has x-intercepts at $(0, 0)$ and $(10, 0)$, so the x-coordinate of the vertex is $\frac{0+10}{2} = 5$.

The y-coordinate of the vertex is $-2(5)^2 + 20(5) - 37 = 13$.

Therefore, the vertex is at $(5, 13)$.

Partial Factoring

A graph of $y = -2x^2 + 20x - 37$ is below. Additional points can be found using the step pattern $-2, -6, -10, \ldots$.



Questions?