MPM2D: Principles of Mathematics

Orthocentre of a Triangle

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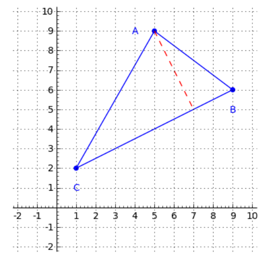
Altitudes

Consider $\triangle ABC$ below.



The line connecting A to BC is called an *altitude*.

Altitudes

An altitude intersects a vertex's opposite side at 90°.

Therefore, the slope of an altitude is the negative reciprocal of the slope of the side with which it intersects.

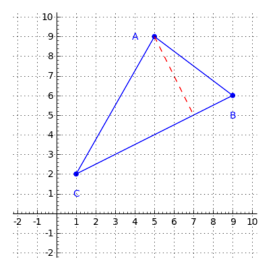
An altitude will not pass through a side's midpoint, unless it is part of an equilateral or isosceles triangle.

Thus, unlike right bisectors and medians, the midpoint does not generally play a role when developing an equation for an altitude.

Altitudes

Example

Determine the equation of the altitude from A in $\triangle ABC$ below.



Altitudes

Determine the slope of BC.

$$\begin{aligned} m_{BC} &= \frac{6-2}{9-1} \\ &= \tfrac{1}{2} \end{aligned}$$

The altitude will have a perpendicular slope of -2.

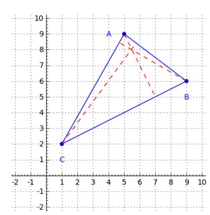
Use the coordinates of vertex A to find the equation.

$$\begin{aligned} 9 &= -2(5) + b \\ b &= 19 \end{aligned}$$

The equation of the altitude is $y = -2x + 19$.

Orthocentre of a Triangle

When all three altitudes are drawn, they intersect at a single point.



Orthocentre of a Triangle

The altitudes from each vertex of a triangle intersect at a point called the orthocentre.

Orthocentre of a Triangle

To find the orthocentre of a triangle, follow the steps below.

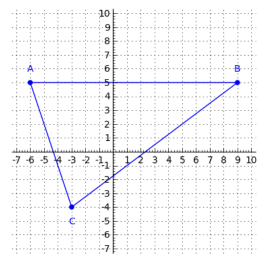
1. Determine the slope of one side.
2. Determine the perpendicular slope to that side.
3. Use the perpendicular slope and the opposite vertex to determine the equation of the altitude from that vertex.
4. Repeat steps 1-3 for another side.
5. Find the point of intersection of the two altitudes.

As always, be on the lookout for shortcuts.

Orthocentre of a Triangle

Example

Determine the location of the orthocentre of the triangle with vertices at $A(-6,5)$, $B(9,5)$ and $C(-3,-4)$.



Orthocentre of a Triangle

Since A and B have the same y-coordinate, AB is a horizontal line segment.

Therefore, the altitude from C is a vertical line with equation $x=-3$.

Next, determine the equation of an altitude from another vertex, such as B.

Determine the slope of AC.

$$\begin{aligned} m_{AC} &= \frac{-4-5}{-3+6} \\ &= -3 \end{aligned}$$

Since the slope of AC is -3, the slope of the altitude is the negative reciprocal, $\frac{1}{3}$.

Orthocentre of a Triangle

Using this slope, along with the coordinates of B, determine the equation of the altitude from B.

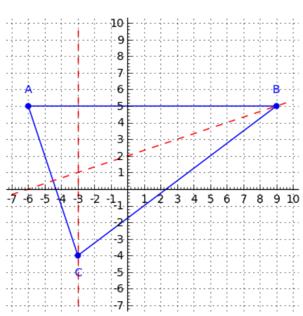
$$\begin{aligned} 5 &= \tfrac{1}{3}(9)+b \\ b &= 2 \\ y &= \tfrac{1}{3}x+2 \end{aligned}$$

Substitute $x=-3$ into this equation to determine the point of intersection of the altitudes.

$$\begin{aligned} y &= \tfrac{1}{3}(-3)+2 \\ y &= 1 \end{aligned}$$

The orthocentre is located at $(-3,1)$.

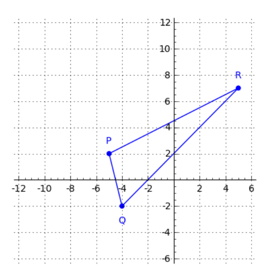
Orthocentre of a Triangle



Orthocentre of a Triangle

Example

Determine the location of the orthocentre of the triangle with vertices at $P(-5,2)$, $Q(-4,-2)$ and $R(5,7)$.



Orthocentre of a Triangle

For the equation of the altitude from P, find the slope of QR.

$$m_{QR} = \frac{-2-7}{-4-5} = 1$$

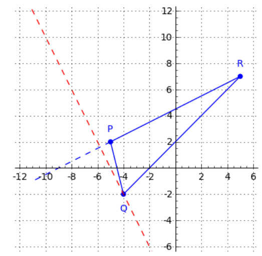
The altitude will have a perpendicular slope of -1.

Use this slope with vertex P to find its equation.

$$2 = -1(-5) + b$$
$$b = -3$$
$$y = -x - 3$$

Orthocentre of a Triangle

The altitude from Q will fall outside of ΔPQR as shown. In this case, PR is extended.



Orthocentre of a Triangle

Find the slope of PR.

$$m_{PR} = \frac{7-2}{5-(-5)} = \frac{1}{2}$$

The altitude will have a perpendicular slope of -2.

Use this slope with vertex Q to find its equation.

$$-2 = -2(-4) + b$$
$$b = -10$$
$$y = -2x - 10$$

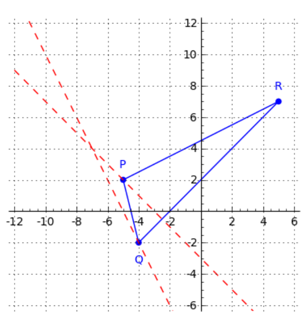
Orthocentre of a Triangle

Find the point of intersection of the two altitudes.

$$-x - 3 = -2x - 10$$
$$x = -7$$
$$y = -(-7) - 3$$
$$y = 4$$

The orthocentre is located at $(-7, 4)$.

Orthocentre of a Triangle



Questions?