MPM2D: Principles of Mathematics

# Factoring Polynomials

## Part 6: Multi-Stage Factoring

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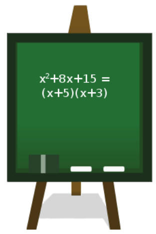






## Factoring Trinomials

**Recap**

Factor $25x^2 + 10x + 1$.

While this complex trinomial can be factored using decomposition, it is a perfect square since $2\sqrt{25}\sqrt{1} = 2 \times 5 \times 1 = 10$.

Therefore, $25x^2 + 10x + 1 = (5x + 1)^2$





## Multi-Stage Factoring

We have covered several types of factoring in this course including:

- common factoring
- factoring by grouping
- factoring simple trinomials
- factoring complex trinomials
- factoring differences of squares
- factoring perfect squares

Sometimes it is necessary to perform more than one type of factoring on a polynomial expression, if it is suitably complex.





## Multi-Stage Factoring

**Example**

Fully factor $2x^2 - 4x - 30$.

Since the expression is a complex trinomial, a possible solution uses decomposition.

$$\begin{aligned} 2x^2 - 4x - 30 &= 2x^2 + 6x - 10x - 30 \\ &= 2x(x+3) - 10(x+3) \\ &= (2x - 10)(x+3) \end{aligned}$$

Note that the first factor contains a common factor of 2, which can be extracted.

$$2x^2 - 4x - 30 = 2(x-5)(x+3)$$





## Multi-Stage Factoring

Common factoring the expression first might be a better solution, since the values of all coefficients and the constant term will be reduced.

$$2x^2 - 4x - 30 = 2(x^2 - 2x - 15)$$

The resulting expression involves a simple trinomial, which is straightforward to factor.

$$2(x^2 - 2x - 15) = 2(x-5)(x+3)$$

This is the same answer as the previous solution. In general, it is a good idea to common factor first.





## Multi-Stage Factoring

**Example**

Factor $10x^3 - 28x^2 - 6x$.

This cubic expression cannot be factored by grouping, but contains a common factor of $2x$ in all three terms.

$$10x^3 - 28x^2 - 6x = 2x(5x^2 - 14x - 3)$$

The quadratic expression inside of the brackets can be factored using decomposition.

$$\begin{aligned} 2x(5x^2 - 14x - 3) &= 2x(5x^2 - 15x + x - 3) \\ &= 2x(5x[x-3] + 1[x-3]) \\ &= 2x(5x+1)(x-3) \end{aligned}$$

## Multi-Stage Factoring

**Example**

Factor $20x^3 - 60x^2 + 45x$.

Common factor $5x$ from all terms.

$$20x^3 - 60x^2 + 45x = 5x(4x^2 - 12x + 9)$$

The expression inside of the brackets is a perfect square, since $2\sqrt{4}\sqrt{9} = 2 \times 2 \times 3 = 12$.

$$5x(4x^2 - 12x + 9) = 5x(2x - 3)^2$$

Remember that the sign in the binomial will match that of the middle term.





## Multi-Stage Factoring

**Example**

Factor $3x^4 - 15x^2 + 18$.

Start by common factoring 3 from all terms.

$$3x^4 - 15x^2 + 18 = 3(x^4 - 5x^2 + 6)$$

Although the expression inside of the brackets is not quadratic, it can be factored using the same techniques.

In this case, the factored expression will have the form $(x^2 + p)(x^2 + q)$ instead of $(x + p)(x + q)$.

$$3(x^4 - 5x^2 + 6) = 3(x^2 - 2)(x^2 - 3)$$





## Multi-Stage Factoring

**Example**

Factor $12x^2 - 27$.

Although the expression looks like a difference of squares, $\sqrt{12}$ and $\sqrt{27}$ are not integers.

Common factoring 3 from each term, however, produces the expression $3(4x^2 - 9)$.

This difference of squares inside of the brackets is factorable.

$$3(4x^2 - 9) = 3(2x + 3)(2x - 3)$$





## Multi-Stage Factoring

**Example**

Factor $16x^4 - 1$.

Since $\sqrt{16} = 4$ and $\sqrt{1} = 1$, we can treat this expression as a difference of squares, but with the form $(ax^2 + c)(ax^2 - c)$ instead.

$$16x^4 - 1 = (4x^2 + 1)(4x^2 - 1)$$

Although the expression has been factored, it can be factored *again*, since $4x^2 - 1$ is *another* difference of squares.

$$(4x^2 + 1)(4x^2 - 1) = (4x^2 + 1)(2x + 1)(2x - 1)$$





## Questions?