MPM2D: Principles of Mathematics

Solving Linear Systems

Solving by Substitution (Part 2)

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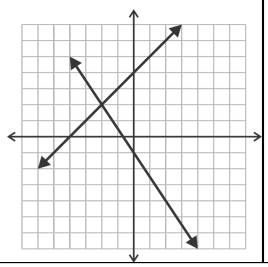

Solving Linear Systems Using Substitution

Recap

Solve the linear system $y = 3x + 15$ and $y = -2x - 5$.

Substitute $3x + 15$ for y in the second equation.

$$\begin{aligned} 3x + 15 &= -2x - 5 \\ 5x &= -20 \\ x &= -4 \end{aligned}$$

Substitute $x = -4$ into the first equation.

$$\begin{aligned} y &= 3(-4) + 15 \\ y &= 3 \end{aligned}$$

Since $-2(-4) - 5 = 3$, the solution is $x = -4$ and $y = 3$.

Solving Linear Systems Using Substitution

What if one the equations is not in slope-intercept form, $y = mx + b$?

Consider the linear system below:

$$\begin{aligned} y &= 2x - 10 \\ 3x + y &= 25 \end{aligned}$$

According to the first equation, y is equivalent to the expression $2x - 10$.

This means that $2x - 10$ can be substituted for y in the second equation, similar to the way in which we did it earlier.

Solving Linear Systems Using Substitution

$$\begin{aligned} 3x + (2x - 10) &= 25 \\ 5x &= 35 \\ x &= 7 \end{aligned}$$

Just like before, once the value of x is known, we can solve for y using one of the two equations.

In this case, the first equation already has y isolated.

$$\begin{aligned} y &= 2(7) - 10 \\ y &= 4 \end{aligned}$$

Since $3(7) + 4 = 25$, the solution is $x = 7$ and $y = 4$.

Solving Linear Systems Using Substitution

Example

Solve the linear system $y = -4x + 3$ and $7x - 5y = -15$.

Substitute $-4x + 3$ for y in the second equation.

$$\begin{aligned} 7x - 5(-4x + 3) &= -15 \\ 7x + 20x - 15 &= -15 \\ 27x &= 0 \\ x &= 0 \end{aligned}$$

Don't be put off if a variable has a value of 0. It's a valid value, just like any other.

Solving Linear Systems Using Substitution

Substitute $x = 0$ into the first equation.

$$\begin{aligned} y &= -4(0) + 3 \\ y &= 3 \end{aligned}$$

Checking the second equation, $7(0) - 5(3) = -15$, so the solution is $x = 0$ and $y = 3$.

Solving Linear Systems Using Substitution

Example

Solve the linear system $y = 5x + 3$ and $5x - y = 8$.

Substitute $5x + 3$ for y in the second equation.

$$\begin{aligned} 5x - (5x + 3) &= 8 \\ -3 &= 8 \end{aligned}$$

Since the equation does not make sense, there is no solution to the linear system.

Note that the second equation is $y = 5x - 8$ when converted to slope-intercept form.

Both lines have the same slopes but different y-intercepts, so they are parallel and distinct.

Solving Linear Systems Using Substitution

Example

Solve the linear system $x + 3y = 2$ and $4x - 5y = 25$.

First, we need to isolate a variable in one equation to obtain an expression to substitute into the other.

In this case, it is probably easiest to isolate x in the first equation.

$$x = 2 - 3y$$

Now, substitute into the second equation.

$$\begin{aligned} 4(2 - 3y) - 5y &= 25 \\ 8 - 12y - 5y &= 25 \\ -17y &= 17 \\ y &= -1 \end{aligned}$$

Solving Linear Systems Using Substitution

This time, we solved for y first instead of x.

It doesn't matter which variable is solved for first, since we can find the value of the other variable using the equations.

Since we isolated x earlier, we can use that equation to solve for y.

$$\begin{aligned} x &= 2 - 3(-1) \\ x &= 5 \end{aligned}$$

Since $4(5) - 5(-1) = 25$, the solution is $x = 5$ and $y = -1$.

Solving Linear Systems Using Substitution

Example

Solve the linear system $2x + 3y = 3$ and $-4x - 3y = 15$.

While it is possible to fully isolate a variable in either of the two equations, doing so will introduce fractions.

This is not really a problem, but an alternative method is to isolate a term in one equation that has the same coefficient of a term in the other equation.

Note that there is a $3y$ and a $-3y$ in each equation. Isolating $3y$ in the first, we get

$$3y = 3 - 2x$$

Solving Linear Systems Using Substitution

Now we can substitute $3 - 2x$ for $3y$ in the second equation.

$$\begin{aligned} -4x - (3 - 2x) &= 15 \\ -4x - 3 + 2x &= 15 \\ -2x &= 18 \\ x &= -9 \end{aligned}$$

Substitute $x = -9$ into $3y = 3 - 2x$.

$$\begin{aligned} 3y &= 3 - 2(-9) \\ 3y &= 21 \\ y &= 7 \end{aligned}$$

Since $-4(-9) - 3(7) = 15$, the solution is $x = -9$ and $y = 7$.

Questions?