MPM2D: Principles of Mathematics

Solving Linear Systems

Solving by Substitution (Part 1)

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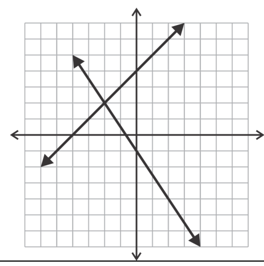
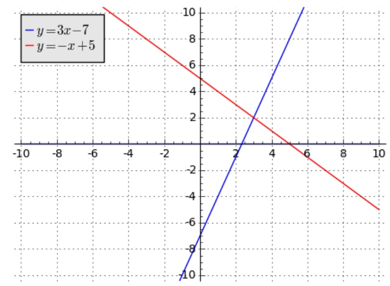


Graphing Linear Systems

Recap

Graph the linear system $y = 3x - 7$ and $y = -x + 5$.



Limitations With Graphing

Graphing is a useful technique, but there are several limitations of solving linear systems graphically.

- It is time-consuming,
- the domain/range may be difficult to predict, or
- points of intersection may be hard to read accurately.

Today, we will examine a method for solving linear systems *algebraically*.

Solving Linear Systems Using Substitution

First, a short detour.

Note that we can express the value 5 in multiple ways:

$$5 = 4 + 1$$
$$5 = 3 + 2$$

In each equation above, we are stating that 5 is equivalent to some other expression (either $4 + 1$ or $3 + 2$).

Note as well that the following equation is also true:

$$4 + 1 = 3 + 2$$

What has happened here is the expression $4 + 1$ has been substituted for the 5 in the second equation.

This technique is known as *substitution*.

Solving Linear Systems Using Substitution

Now let's return to the earlier linear system:

$$y = 3x - 7$$
$$y = -x + 5$$

Substituting $3x - 7$ for the y in the second equation gives the following new equation:

$$3x - 7 = -x + 5$$

Notice how the y variable has disappeared completely, leaving us with only one unknown for which we can solve.

$$3x - 7 = -x + 5$$
$$4x = 12$$
$$x = 3$$

Solving Linear Systems Using Substitution

We now know that the solution to the linear system occurs when $x = 3$, but what about the value of y?

Use either of the two equations (whichever is easiest) to solve for y when $x = 3$.

$$y = 3x - 7$$
$$y = 3(3) - 7$$
$$y = 2$$

Therefore, the solution to the linear system is $x = 3$ and $y = 2$.

The point of intersection of the two lines is $(3, 2)$.

Solving Linear Systems Using Substitution

To check if our solution is correct, substitute either x or y into the *other* equation and see if produces the same value.

It is important to use the other equation, since any errors introduced by solving in the first equation are less likely to be replicated in the second.

$$\begin{aligned} y &= -x + 5 \\ y &= -(3) + 5 \\ y &= 2 \end{aligned}$$

Since we obtain the same value for y, our solution is probably correct.

Solving Linear Systems Using Substitution

Example

Solve the linear system $y = 5x - 11$ and $y = 3x + 17$.

Substitute $5x - 11$ for y in the second equation.

$$\begin{aligned} 5x - 11 &= 3x + 17 \\ 2x &= 28 \\ x &= 14 \end{aligned}$$

Substitute $x = 14$ into the second equation.

$$\begin{aligned} y &= 3(14) + 17 \\ y &= 42 + 17 \\ y &= 59 \end{aligned}$$

Solving Linear Systems Using Substitution

The solution appears to be $x = 14$ and $y = 59$. Let's check the first equation to be sure.

$$\begin{aligned} y &= 5(14) - 11 \\ y &= 70 - 11 \\ y &= 59 \end{aligned}$$

Thus, the solution to the linear system is $x = 14$ and $y = 59$.

Note that while it would be possible to solve this system by graphing, the value of y is fairly large and may not easily fit onto an accurate graph.

Solving Linear Systems Using Substitution

Example

Solve the linear system $y = 2x + 5$ and $y = -4x + 12$.

Substitute $2x + 5$ for y in the second equation.

$$\begin{aligned} 2x + 5 &= -4x + 12 \\ 6x &= 7 \\ x &= \tfrac{7}{6} \end{aligned}$$

Substitute $x = \frac{7}{6}$ into the first equation.

$$\begin{aligned} y &= 2\left(\tfrac{7}{6}\right) + 5 \\ y &= \tfrac{7}{3} + 5 \\ y &= \tfrac{22}{3} \end{aligned}$$

Solving Linear Systems Using Substitution

The solution appears to be $x = \frac{7}{6}$ and $y = \frac{22}{3}$. Check using the second equation.

$$\begin{aligned} y &= -4\left(\tfrac{7}{6}\right) + 12 \\ y &= -\tfrac{14}{3} + 12 \\ y &= \tfrac{22}{3} \end{aligned}$$

Thus, the solution to the linear system is $x = \frac{7}{6}$ and $y = \frac{22}{3}$.

In this case, the values of x and y would be dfficult to read accurately from a graph, so the algebraic approach is better suited.

Solving Linear Systems Using Substitution

Example

Solve the linear system $y = 7x + 3$ and $y = 7x - 8$.

Substitute $7x + 3$ for y in the second equation.

$$\begin{aligned} 7x + 3 &= 7x - 8 \\ 0 &= 11 \end{aligned}$$

The result does not make any sense. This is because the two lines are parallel (same slope), but not coincident (different y-intercepts).

Therefore, there are no solutions to this linear system. Remember to check!

Solving Linear Systems Using Substitution

Example

Solve the linear system $y = -\frac{1}{3}x - \frac{1}{2}$ and $y = \frac{1}{6}x - \frac{9}{4}$.

Substitute $-\frac{1}{3}x - \frac{1}{2}$ for y in the second equation.

$$-\frac{1}{3}x - \frac{1}{2} = \frac{1}{6}x - \frac{9}{4}$$

Find a common denominator for all terms.

$$-\frac{4}{12}x - \frac{6}{12} = \frac{2}{12}x - \frac{27}{12}$$

Multiply both sides of the equation by 12 to cancel the denominator, then solve for x.

$$\begin{aligned} -4x - 6 &= 2x - 27 \\ 6x &= 21 \\ x &= \frac{7}{2} \end{aligned}$$

Solving Linear Systems Using Substitution

Substitute $x = \frac{7}{2}$ into the first equation.

$$\begin{aligned} y &= -\frac{1}{3}\left(\frac{7}{2}\right) - \frac{1}{2} \\ y &= -\frac{7}{6} - \frac{1}{2} \\ y &= -\frac{5}{3} \end{aligned}$$

Check using the second equation.

$$\begin{aligned} y &= \frac{1}{6}\left(\frac{7}{2}\right) - \frac{9}{4} \\ y &= \frac{7}{12} - \frac{9}{4} \\ y &= -\frac{5}{3} \end{aligned}$$

Therefore, the solution is $x = \frac{7}{2}$ and $y = -\frac{5}{3}$.

Questions?