MPM2D: Principles of Mathematics

Applications of Linear Systems

Problems Involving Speed, Distance and Time

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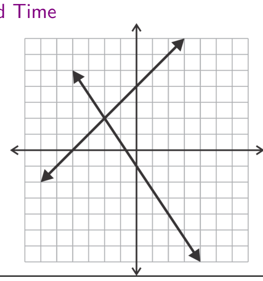

Applications of Linear Systems

Recap

A 200 kg mixture of nuts, sold for \$3.50/kg, is made by combining peanuts (\$2.75/kg) and cashews (\$5.25/kg). How many kilograms of each type of nut is in the mixture?

Let p and c be the masses of the peanuts and cashews.

The two equations are $p + c = 200$ and $2.75p + 5.25c = 700$, since $200 \times 3.50 = 700$.

Using substitution:

$$\begin{aligned} 2.75(200 - c) + 5.25c &= 700 \\ 550 - 2.75c + 5.25c &= 700 \\ c &= 60 \end{aligned}$$

There are 60 kg of cashews, and $200 - 60 = 140$ kg of peanuts in the mixture.

Speed, Distance and Time Problems

Many questions involving speed, distance and time can be solved using linear systems.

There are three ways to relate speed, distance and time.

Speed, Distance and Time

If s, d and t represent speed, distance and time, then $d = st$, $s = \frac{d}{t}$ and $t = \frac{d}{s}$.

Each relationship can be rearranged to create the other two, so there is no need to memorize all three.

Speed, Distance and Time Problems

Example

Two trains, 390 km apart, drive toward each other on parallel tracks. The first train has an average speed of 75 km/h, and the second a speed of 55 km/h. After how long will the trains meet each other? How far from each station will they be?

Let d be the distance travelled by the train moving at 75 km/h, and t the time when it meets the other train.

At this time, t, the other train will have travelled a distance of $390 - d$ km.

This gives the following system of equations:

$$\begin{aligned} d &= 75t \\ 390 - d &= 55t \end{aligned}$$

Speed, Distance and Time Problems

Substitute $75t$ into d in the second equation and solve for t.

$$\begin{aligned} 390 - 75t &= 55t \\ 130t &= 390 \\ t &= 3 \end{aligned}$$

The trains will meet at 3 hours.

At 3 hours, the faster train will have gone $3 \times 75 = 225$ km, while the slower train will have gone $3 \times 55 = 165$ km.

Speed, Distance and Time Problems

Note that the previous example could have also been solved logically, rather than using a linear system.

Since the trains are moving toward each other at 75 km/h and 55 km/h respectively, they are getting closer at an effective speed of $75 + 55 = 130$ km/h.

Since $\frac{390}{130} = 3$, the trains must meet at 3 hours.

The solutions to many problems can be deduced using similar arguments. Use them when appropriate.

Speed, Distance and Time Problems

Example

A current flows through a river at a constant rate. A canoeist paddles 6 km upstream (against the current) in 3 hours. The return trip downstream (with the current) takes 2 hours. Determine the speed of the canoeist in still water, and the speed of the current.

Let c be the speed of the canoeist, and r the speed of the river current.

When the canoeist is travelling upstream, his/her speed is *reduced*. Thus, the canoeist travels upstream with an effective speed of $c - r$.

When travelling downstream, the canoeist's speed is *increased* to $c + r$.

Speed, Distance and Time Problems

Using the relationship $s = \frac{d}{t}$, we can create the following linear system representing the journeys upstream and downstream:

$$\begin{aligned} c - r &= \tfrac{6}{3} & \rightarrow \quad c - r &= 2 \\ c + r &= \tfrac{6}{2} & \rightarrow \quad c + r &= 3 \end{aligned}$$

Add and subtract the two equations to solve for c and r.

$$\begin{array}{rl} & c - r = 2 \\ - & c + r = 3 \\ \hline & -2r = -1 \\ & r = \tfrac{1}{2} \end{array} \qquad \begin{array}{rl} & c - r = 2 \\ + & c + r = 3 \\ \hline & 2c \quad = 5 \\ & c \quad = \tfrac{5}{2} \end{array}$$

The speed of the canoeist is 2.5 km/h, and the speed of the current is 0.5 km/h.

Speed, Distance and Time Problems

Example

A student lives 11 km from school. On her morning commute, she walks to a bus stop at an average speed of 6 km/h. She then rides the bus for the remainder of the distance, at an average speed of 45 km/h. If the entire trip takes 32 minutes, what are the times spent walking and riding the bus?

Let d be the distance walked, and t the time spent walking.

Thus, an equation representing the walking portion of the trip is $d = 6t$.

Speed, Distance and Time Problems

Since the total distance is 11 km, the student rides the bus for $11 - d$ km.

Similarly, since the total time is 32 minutes (or $\frac{8}{15}$ of an hour), she rides the bus for $\frac{8}{15} - t$ hours.

Note that we could have worked in minutes instead, which would have required converting from km/h to km/min first.

An equation representing the portion of the trip spent riding the bus is $11 - d = 45\left(\frac{8}{15} - t\right)$.

Therefore, we have the following linear system to solve:

$$\begin{aligned} d &= 6t \\ 11 - d &= 45\left(\tfrac{8}{15} - t\right) \end{aligned}$$

Speed, Distance and Time Problems

Substitute $6t$ into d in the second equation and solve for t.

$$\begin{aligned} 11 - 6t &= 45\left(\tfrac{8}{15} - t\right) \\ 11 - 6t &= 24 - 45t \\ 39t &= 13 \\ t &= \tfrac{1}{3} \end{aligned}$$

The student spends $\frac{1}{3}$ of an hour (20 minutes) walking.

If she walks for $\frac{1}{3}$ of an hour, the time spent riding the bus is $\frac{8}{15} - \frac{1}{3} = \frac{1}{5}$ of an hour (12 minutes).

Questions?