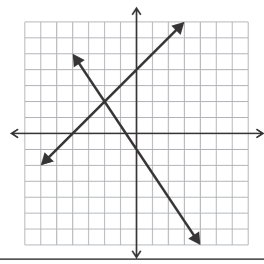
MPM2D: Principles of Mathematics

Applications of Linear Systems

Numeric and Value Problems

J. Garvin



Solving Linear Systems

Recap

Solve the linear system $9x + 5y = 1$ and $12x + 25y = 38$.

Eliminate the x terms by multiplying the first equation by 4 and the second by 3.

Eliminate the y terms by multiplying the first equation by 5 (leave the second as is).

$$\begin{array}{rrl} & 36x + 75y = & 114 \\ - & 36x + 20y = & 4 \\ \hline & 55y = & 110 \\ & y = & 2 \end{array} \qquad \begin{array}{rrl} & 45x + 25y = & 5 \\ - & 12x + 25y = & 38 \\ \hline & 33x \quad = & -33 \\ & x \quad = & -1 \end{array}$$

The solution is $x = -1$ and $y = 2$.

Applications of Linear Systems

Many problems can be modelled by two or more linear relations, forming a linear system.

Any situaion that involves a linear system can be solved using the three techniques (graphing, substitution, elimination) we have covered.

The most common applications involve:

- numeric or value-based problems,
- mixtures and percentages,
- times to complete one or more tasks, or
- problems relating speed, distance and time.

Today, we will look at the first of these four types.

Numeric Applications

Example

Two numbers have a sum of 26. When the smaller number is tripled, it is two greater than the larger number. Determine the values of the two numbers.

The first thing to do when dealing with word problems like these is to *define* the two variables being used.

This will help us to create equations involving the two variables, and to interpret the solution, if one exists.

Let S be the smaller number, and L the larger number.

From the first sentence, we know that $S + L = 26$.

From the second, we know that $3S - L = 2$, or $L = 3S - 2$.

Numeric Applications

Since L is isolated, substitution might be a good method of solving this linear system.

Substitute $3S - 2$ for L in $S + L = 26$ and solve for S.

$$\begin{aligned} S + (3S - 2) &= 26 \\ 4S &= 28 \\ S &= 7 \end{aligned}$$

Since $L = 3S - 2$, $L = 3(7) - 2 = 19$.

Therefore, the smaller value is 7 and the larger value is 19.

This can be verified by checking that $7 + 19 = 26$, as per the other equation.

Value Applications

Example

A handful of quarters and dimes contains 21 coins, for a total of \$3.30. Determine the number of each coin.

Let Q be the number of quarters and D the number of dimes.

Since there are 21 coins, $Q + D = 21$, or $Q = 21 - D$.

Quarters have a value of 25¢, and dimes 10¢.

A second equation, relating their values to the total amount, is $0.25Q + 0.10D = 3.30$, or $25Q + 10D = 330$.

Value Applications

Substitute $21 - D$ for Q in $25Q + 10D = 330$.

$$\begin{aligned} 25(21 - D) + 10D &= 330 \\ 525 - 25D + 10D &= 330 \\ -15D &= -195 \\ D &= 13 \end{aligned}$$

Since $Q = 21 - D$, $Q = 21 - 13 = 8$.

Therefore, there are 8 quarters and 13 dimes.

Checking, $25(8) + 10(13) = 330$.

Value Applications

Example

A retailer orders 40 t-shirts and 21 pairs of jeans from a supplier for \$289.00. The next week, she orders 28 t-shirts and 17 pairs of jeans for \$223.00. How much does the supplier charge for each item?

Let t be the cost of a t-shirt, and j the cost of a pair of jeans.

From the first sentence, we know that $40t + 21j = 289$.

From the second, we know that $28t + 17j = 223$.

In this case, it is proably easier to use elimination to solve the linear system.

Value Applications

Multiply the first equation by 7 and the second equation by 10 to eliminate the t terms.

$$\begin{aligned} 280t + 147j &= 2023 \\ -\quad 280t + 170j &= 2230 \\ \hline -23j &= -207 \\ j &= 9 \end{aligned}$$

Substitute $j = 9$ into the first equation.

$$\begin{aligned} 40t + 21(9) &= 289 \\ 40t &= 100 \\ t &= 2.5 \end{aligned}$$

A t-shirt costs \$2.50 and a pair of jeans costs \$9.00.

Questions?