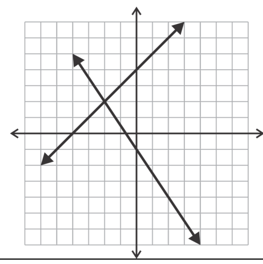
MPM2D: Principles of Mathematics

Solving Linear Systems

Solving by Graphing

J. Garvin



Graphing Linear Systems

Previously, you have graphed linear relations, usually having been given information about its *slope* and its *y-intercept*.

You have also solved word problems involving linear relations.

In this unit, we will investigate problems that involve *systems of linear equations*, or simply *linear systems*.

Systems of Equations

A system of equations is a series of two or more equations with the same set of unknowns.

Linear Systems

A linear system is a system of equations in which all equations represent linear relations.

Graphing Linear Systems

Example

Graph the linear system defined by the two equations $y = 2x - 4$ and $y = -x + 8$.

The first equation has a y-intercept of -4 and a slope of 2.

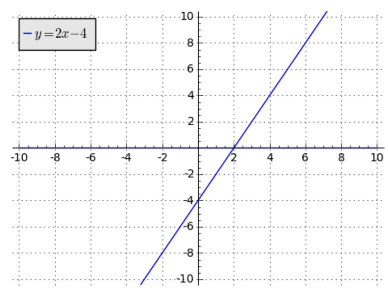
The rise is 2 and the run is 1, since $2 = \frac{2}{1}$.

Starting at -4 on the y-axis, move up two units, then right one unit, since the slope is positive.

Repeat as necessary until the graph of $y = 2x - 4$ is drawn.

Graphing Linear Systems

The graph of $y = 2x - 4$ is shown below.



Graphing Linear Systems

The second equation's y-intercept is 8 and its slope is -1.

The slope is not 0 (or -0 as it may be), since $y = 0x + 8$ is the same as $y = 8$, a horizontal line.

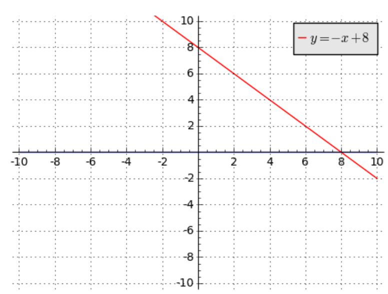
The rise is 1 and the run is 1, since $-1 = -\frac{1}{1}$.

Starting at 8 on the y-axis, move down one unit, then right one unit, since the slope is negative.

Repeat as necessary until the graph of $y = -x + 8$ is drawn.

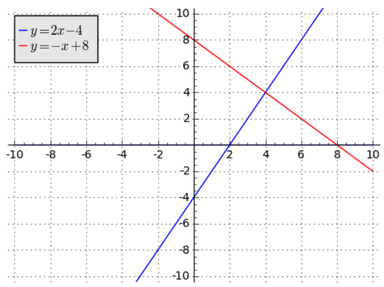
Graphing Linear Systems

The graph of $y = -x + 8$ is shown below.



Graphing Linear Systems

The graph of both lines, together, is below.



Solving Linear Systems Graphically

In the previous example, the two lines *intersect* at $(4, 4)$.

This means that $(4, 4)$ is a solution for both equations.

$$\begin{aligned} y &= 2x - 4 \\ &= 2(4) - 4 \\ &= 4 \end{aligned} \qquad \begin{aligned} y &= -x + 8 \\ &= -(4) + 8 \\ &= 4 \end{aligned}$$

To *solve* a linear system is to find the values of x and y, such that they satisfy both equations.

Solution of a Linear System

Graphically, the solution to a linear system is the *point of intersection* (POI) of the two lines.

Solving Linear Systems Graphically

Example

Solve the linear system defined by the two equations $y = x + 2$ and $y = \frac{2}{3}x$.

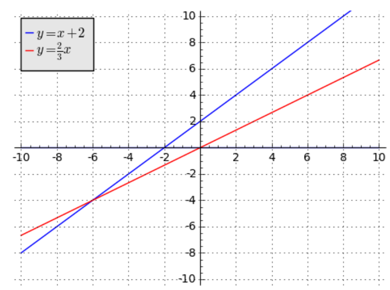
The first equation has a y-intercept of 2 and a slope of 1.

The second equation's y-intercept is 0, since there is no value provided. Its slope is $\frac{2}{3}$.

Since the two lines *diverge* (get further apart) as x increases, it is important to extend the lines left for smaller values of x to find the point of intersection.

Solving Linear Systems Graphically

The graph of the linear system is below.



The solution is $(-6, -4)$, or $x = -6$ and $y = -4$.

Solving Linear Systems Graphically

Example

Solve the linear system defined by the two equations $x - 2y = -6$ and $2x + y = 8$.

This time, the linear relations are expressed in *standard form*.

Recall that standard form of a linear relation is $Ax + By = C$, where A, B and C are integers, and $A > 0$.

There are two common ways to graph a linear relation in standard form:

- convert the relation to slope-intercept form, $y = mx + b$, or
- determine the x- and y-intercepts of the relation.

Solving Linear Systems Graphically

y-intercepts occur when $x = 0$.

$$\begin{aligned} (0) - 2y &= -6 \\ y &= 3 \end{aligned} \qquad \begin{aligned} 2(0) + y &= 8 \\ y &= 8 \end{aligned}$$

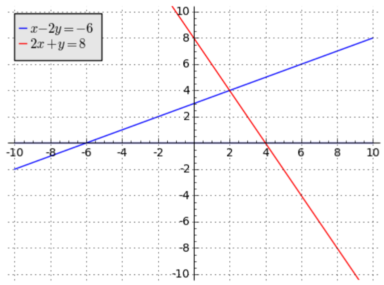
x-intercepts occur when $y = 0$.

$$\begin{aligned} x - 2(0) &= -6 \\ x &= -6 \end{aligned} \qquad \begin{aligned} 2x + 0 &= 8 \\ x &= 4 \end{aligned}$$

Therefore, the first equation has intercepts at $(0, 3)$ and $(-6, 0)$, while the second has intercepts at $(0, 8)$ and $(4, 0)$.

Solving Linear Systems Graphically

The graph of the linear system is below.



The solution is $(2,4)$, or $x=2$ and $y=4$.

Solving Linear Systems Graphically

Example

Solve the linear system defined by the two equations $y=-\frac{1}{2}x-2$ and $2x+4y=12$.

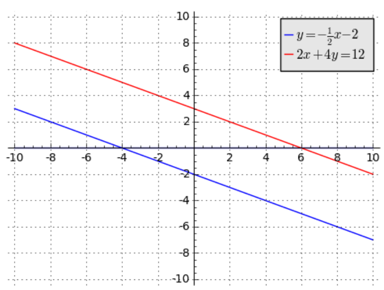
One equation is in slope-intercept form, while the other is in standard form.

The first equation has a slope of $-\frac{1}{2}$ and a y-intercept of -2.

The second equation has intercepts at $(6,0)$ and $(0,3)$, since $2(6)+4(0)=12$ and $2(0)+4(3)=12$.

Solving Linear Systems Graphically

The graph of the linear system is below.



Where is the point of intersection?

Number of Solutions

In the last example, there was no solution to the linear system because the two lines were parallel.

A linear system with no solution is *inconsistent*.

When solving linear systems involving two variables, there are three possible outcomes.

Number of Solutions of a Linear System

A linear system may have:

- 1 unique solution if the two lines are not parallel,
- 0 solutions if the two lines are parallel and *distinct*, or
- infinite solutions if the two lines are *coincident*.

Recall that two lines are parallel if they have the same slope. This is usually a good thing to check before solving.

Number of Solutions

Example

Determine the number of solutions of the linear system given by $y=\frac{3}{2}x+1$ and $3x-2y=10$.

The second equation can be converted to slope-interept form so that it can be compared to the first equation.

$$-2y=-3x+10$$
$$y=\frac{3}{2}x-5$$

The slopes are the same, so the lines are parallel or coincident.

Since the lines have different y-intercepts, they must be parallel. Therefore, there are no solutions and the linear system is inconsistent.

Solving Linear Systems Graphically

Example

Determine the number of solutions of the linear system given by $y=\frac{5}{2}x+3$ and $5x-2y=-6$.

Again, convert the second equation to slope-interept for comparison.

$$-2y=-5x-6$$
$$y=\frac{5}{2}x+3$$

Since the two equations are the same, the two lines are coincident. There are an infinite number of solutions.

Questions?