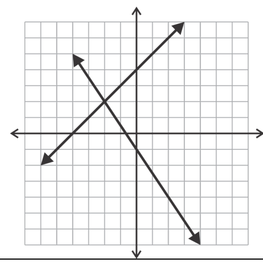
MPM2D: Principles of Mathematics

Solving Linear Systems

Solving by Elimination (Part 2)

J. Garvin



Solving Linear Systems Using Elimination

Recap

Solve the linear system $2x - 12y = 9$ and $7x + 12y = 18$.

$12y$ and $-12y$ have different signs, so add the equations.

$$\begin{array}{rrcl} & 2x - 12y & = & 9 \\ + & 7x + 12y & = & 18 \\ \hline & 9x & = & 27 \\ & x & = & 3 \end{array}$$

Substitute $x = 3$ into the first equation.

$$\begin{aligned} 2(3) - 12y &= 9 \\ -12y &= 3 \\ y &= -\tfrac{1}{4} \end{aligned}$$

$7(3) + 12\left(-\frac{1}{4}\right) = 18$, so the solution is $x = 3$ and $y = -\frac{1}{4}$.

Solving Linear Systems Using Elimination

All of the previous examples that used elimination involved terms with similar coefficients.

Elimination can also be used when there are no terms with similar coefficients.

Recall that we can perform the same operation to both sides of an equation, and maintain its "balance".

For example, multiplying both sides of $x + 3y = 7$ by 2 gives the new equation $2x + 6y = 14$.

Both $x + 3y = 7$ and $2x + 6y = 14$ are *equivalent* equations, since all pairs of x and y that satisfy the first equation also satisfy the second.

Solving Linear Systems Using Elimination

Consider the linear system $2x + 3y = 11$ and $4x + 5y = 21$.

No terms have similar coefficients, but multiplying the first equation by 2 gives the equivalent equation $4x + 6y = 22$.

Now we can subtract the new equation from the second.

$$\begin{array}{rrcl} & 4x + 6y & = & 22 \\ - & 4x + 5y & = & 21 \\ \hline & y & = & 1 \end{array}$$

Substituting $y = 1$ into the first equation,

$$\begin{aligned} 2x + 3(1) &= 11 \\ 2x &= 8 \\ x &= 4 \end{aligned}$$

Since $4(4) + 5(1) = 21$, the solution is $x = 4$ and $y = 1$.

Solving Linear Systems Using Elimination

Example

Solve the linear system $3x + 2y = 18$ and $9x + 4y = 60$.

Multiply the first equation by 3 then subtract.

$$\begin{array}{rrcl} & 9x + 6y & = & 54 \\ - & 9x + 4y & = & 60 \\ \hline & 2y & = & -6 \\ & y & = & -3 \end{array}$$

Substitute $y = -3$ into the first equation.

$$\begin{aligned} 3x + 2(-3) &= 18 \\ 3x &= 24 \\ x &= 8 \end{aligned}$$

$9(8) + 4(-3) = 60$, so the solution is $x = 8$ and $y = -3$.

Solving Linear Systems Using Elimination

Example

Solve the linear system $10x - 25y = 11$ and $15x + 5y = 8$.

Multiply the second equation by 5 then add.

$$\begin{array}{rrcl} & 75x + 25y & = & 40 \\ + & 10x - 25y & = & 11 \\ \hline & 85x & = & 51 \\ & x & = & \frac{3}{5} \end{array}$$

Substitute $x = \frac{3}{5}$ into the first equation.

$$\begin{aligned} 10\left(\tfrac{3}{5}\right) - 25y &= 11 \\ -25y &= 5 \\ y &= -\tfrac{1}{5} \end{aligned}$$

$15\left(\frac{3}{5}\right) + 5\left(-\frac{1}{5}\right) = 8$, so the solution is $x = \frac{3}{5}$ and $y = -\frac{1}{5}$.

Solving Linear Systems Using Elimination

Example

Solve the linear system $3x - 4y = -34$ and $2x + 3y = 17$.

In this situation, multiplying the terms in one equation does not work unless we introduce fractions (e.g. multiply the second equation by $\frac{3}{2}$ to create a $3x$ term).

An alternative method is to multiply both equations, each by a different value, to create similar coefficients.

Here, we can multiply the first equation by 2 and the second by 3, creating a $6x$ term in each.

$$\begin{aligned} 3x - 4y &= -34 \quad \times 2 \rightarrow \quad 6x - 8y = -68 \\ 2x + 3y &= 17 \quad \times 3 \rightarrow \quad 6x + 9y = 51 \end{aligned}$$

Solving Linear Systems Using Elimination

Now subtract the two new equations.

$$\begin{array}{rr} & 6x - 8y = -68 \\ - & 6x + 9y = 51 \\ \hline & -17y = -119 \\ & y = 7 \end{array}$$

Substitute $y = 7$ into the second equation.

$$\begin{aligned} 2x + 3(7) &= 17 \\ 2x &= -4 \\ x &= -2 \end{aligned}$$

$3(-2) - 4(7) = -34$, so the solution is $x = -2$ and $y = 7$.

Solving Linear Systems Using Elimination

Since elimination produces a short equation involving one variable (e.g. $2x = 8$), it is may be desirable to use only elimination and avoid substitution altogether.

To use this "double elimination" method, use elimination on one of the variables, then do the same for the second.

It may be necessary to multiply one or both equations before this can be done.

Solving Linear Systems Using Elimination

Example

Solve the linear system $3x - 10y = -27$ and $6x + 15y = 51$.

To eliminate the x terms, multiply the first equation by 2 and subtract since the signs are the same.

$$\begin{array}{rr} & 6x - 20y = -54 \\ - & 6x + 15y = 51 \\ \hline & -35y = -105 \\ & y = 3 \end{array}$$

From here, we could substitute $y = 3$ into one of the equations, but let's try using elimination again for the y terms.

Solving Linear Systems Using Elimination

To eliminate the y terms, multiply the first equation by 3 and the second by 2. Add since the signs are different.

$$\begin{array}{rr} & 9x - 30y = -81 \\ + & 12x + 30y = 102 \\ \hline & 21x \qquad = 21 \\ & x \qquad = 1 \end{array}$$

The solution to the linear system is $x = 1$ and $y = 3$. You can verify this by checking both equations.

Questions?