MPM2D: Principles of Mathematics

Solving Linear Systems

Solving by Elimination (Part 1)

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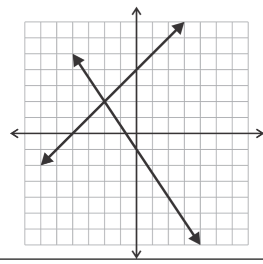

Solving Linear Systems Using Substitution

Recap

Solve the linear system $5x + 2y = 36$ and $3x + 2y = 20$.

Isolate $2y$ in the first equation.

$$2y = 36 - 5x$$

Substutute $36 - 5x$ for $2y$ in the second equation.

$$\begin{aligned} 3x + (36 - 5x) &= 20 \\ -2x &= -16 \\ x &= 8 \end{aligned}$$

Substitute $x = 8$ into $2y = 36 - 5x$.

$$\begin{aligned} 2y &= 36 - 5(8) \\ 2y &= -4 \\ y &= -2 \end{aligned}$$

Solving Linear Systems Using Substitution

In the previous example, the solution is $x = 8$ and $y = -2$.

Solving for the first unknown requires substituting an expression into an equation, then using the distributive property to expand and simplify the new equation.

There is another technique that we will look at today that can avoid this requirement.

Solving Linear Systems Using Elimination

First, a short detour.

Consider the following two equations:

$$\begin{aligned} 7 &= 1 + 6 \\ 5 &= 1 + 4 \end{aligned}$$

Now, let's subtract each column of values:

$$2 = 0 + 2$$

Note that the resulting new equation is true, and that the two identical values (1) have been eliminated.

This technique is called *elimination*.

Solving Linear Systems Using Elimination

Let's return to the earlier linear system, and subtract each column of terms:

$$\begin{array}{rl} & 5x + 2y = 36 \\ - & 3x + 2y = 20 \\ \hline & 2x \quad\;\; = 16 \end{array}$$

Notice how y has been eliminated from the equations, leaving a simple equation involving only one unknown.

$$\begin{aligned} 2x &= 16 \\ x &= 8 \end{aligned}$$

Solving Linear Systems Using Elimination

Now it is possible to solve for the value of the second unknown as before, by substituting $x = 8$ into either of the two equations.

$$\begin{aligned} 3(8) + 2y &= 20 \\ 2y &= -4 \\ y &= -2 \end{aligned}$$

Since $5(8) + 2(-2) = 36$, the solution to the linear system is $x = 8$ and $y = -2$.

Both substitution and elimination will produce the same solution, if all steps are carried out correctly.

Solving Linear Systems Using Elimination

Example

Solve the linear system $9x - 5y = -24$ and $9x - 2y = -15$.

Arrange the two equations vertically and subtract.

$$\begin{array}{rr} & 9x - 5y = -24 \\ - & 9x - 2y = -15 \\ \hline & -3y = -9 \\ & y = 3 \end{array}$$

In this case, the x variables were eliminated since the coefficients were the same.

Solving Linear Systems Using Elimination

Substitute $y = 3$ into one of the equations.

$$\begin{aligned} 9x - 2(3) &= -15 \\ 9x &= -9 \\ x &= -1 \end{aligned}$$

Since $9(-1) - 5(3) = -24$, the solution is $x = -1$ and $y = 3$.

Solving Linear Systems Using Elimination

Example

Solve the linear system $3x + 8y = 10$ and $5x - 8y = 6$.

In this case, subtracting does not eliminate any variables.

$$\begin{array}{rr} & 3x + \ 8y = 10 \\ - & 5x - \ 8y = 6 \\ \hline & -2x + 16y = 4 \end{array}$$

Since the coefficients differ only in terms of their sign, *add* the two equations instead.

$$\begin{array}{rr} & 3x + 8y = 10 \\ + & 5x - 8y = 6 \\ \hline & 8x \quad\quad = 16 \\ & x \quad\quad = 2 \end{array}$$

Solving Linear Systems Using Elimination

Substitute $x = 2$ into one of the equations.

$$\begin{aligned} 3(2) + 8y &= 10 \\ 8y &= 4 \\ y &= \tfrac{1}{2} \end{aligned}$$

Since $5(2) - 8\left(\frac{1}{2}\right) = 6$, the solution is $x = 2$ and $y = \frac{1}{2}$.

We have now seen two ways to eliminate terms from two linear equations.

Eliminating Terms Based On Their Signs

When using elimination, *subtract* one equation from the other if the coefficients of the similar terms have the *same* sign, and *add* one equation to the other if the signs are *different*.

Solving Linear Systems Using Elimination

Example

Solve the linear system $4x + 3y = 7$ and $4x + 3y = 2$.

Subtract the equations.

$$\begin{array}{rr} & 4x + 3y = 7 \\ - & 4x + 3y = 2 \\ \hline & 0 = 5 \end{array}$$

Since this is not possible, there is no solution to the linear system. The two lines are parallel and distinct.

Questions?