MPM2D: Principles of Mathematics

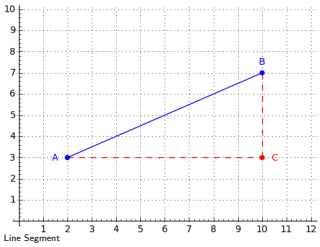
Length of a Line Segment

J. Garvin

Length of a Line Segment

Investigate

Plot the points $A(2,3)$ and $B(10,7)$, and draw line segment AB. Find a third point C such that line segments AC and BC make a right triangle with AB as the hypotenuse. How long are AC and BC? How long is AB?



Length of a Line Segment

In the diagram, AC is 8 units long, while BC is 4 units long.

The notation $|PQ|$ is often used to indicate the *magnitude* (size) of a line segment PQ.

Thus, we can say $|AC| = 8$ and $|BC| = 4$.

Use the Pythagorean Theorem to calculate the length of AB.

$$|AB|^2 = |AC|^2 + |BC|^2$$
$$|AB|^2 = 8^2 + 4^2$$
$$|AB|^2 = 80$$
$$|AB| = \sqrt{80}$$
$$|AB| = 4\sqrt{5}$$
$$|AB| \approx 8.94$$

Length of a Line Segment

There is an algebraic method of finding the length of a line segment, which does not involve graphing.

Recall that AC is a horizontal line segment, and that $|AC| = 8$.

The x-coordinates of $A(2,3)$ and $C(10,3)$ are 10 and 2 respectively.

The length of AC is the difference in the x-coordinates – that is, $10 - 2 = 8$.

Similarly, for the vertical line segment BC, the y-coordinates of $B(10,7)$ and $C(10,3)$ are 7 and 3, with a difference of $7 - 3 = 4$.

Length of a Line Segment

This gives us the following two relationships for lengths of horizontal and vertical line segments.

Length of a Horizontal Line Segment

The length of the horizontal line segment PQ connecting $P(x_1, y_1)$ and $Q(x_2, y_2)$, is given by $|PQ| = x_2 - x_1$.

Length of a Vertical Line Segment

The length of the vertical line segment PQ connecting $P(x_1, y_1)$ and $Q(x_2, y_2)$, is given by $|PQ| = y_2 - y_1$.

Length of a Line Segment

Example

Determine $|JK|$ for $J(5,-2)$ and $K(11,-2)$.

Since the y-coordinates are the same for both J and K, the line segment is horizontal.

Therefore, $|JK| = 11 - 5 = 6$ units.

Length of a Line Segment

Example

Determine the value of k if $|EF| = 12$ for $E(-3, 1)$ and $F(-3, k)$.

Since the x-coordinates are the same for both E and F, the line segment is vertical.

If $|EF| = 12$, then $12 = k - 1$ units, so $k = 13$.

Note that if $k = -11$, then $|EF| = 1 - k = 1 - (-11) = 12$ as well.

Therefore, k can be either 13 or -11.

Length of a Line Segment

Returning to the investigation, how can we determine the length of the hypotenuse AB algebraically?

The Pythagorean Theorem can be used if we know the rise and the run of the right triangle.

Since the rise and run are vertical and horizontal line segments, we can use the formulae developed earlier.

$$|AB|^2 = |AC|^2 + |BC|^2$$
$$|AB|^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Length of a Line Segment

This gives us a general formula for the length of a line segment.

Length of a Line Segment

The length of the line segment PQ connecting $P(x_1, y_1)$ and $Q(x_2, y_2)$, is given by $|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Sometimes, the length of a line segment is phrased "the distance between two points", and the formula above nicknamed the Distance Formula.

Length of a Line Segment

Example

Verify that $|AB| = 4\sqrt{5}$ for $A(2, 3)$ and $B(10, 7)$.

$$|AB| = \sqrt{(10 - 2)^2 + (7 - 3)^2}$$
$$|AB| = \sqrt{8^2 + 4^2}$$
$$|AB| = \sqrt{80}$$
$$|AB| = 4\sqrt{5}$$

Length of a Line Segment

Example

Determine the distance between the points $G(1, -4)$ and $H(-6, -3)$.

$$|GH| = \sqrt{(-6 - 1)^2 + (-3 - (-4))^2}$$
$$|GH| = \sqrt{(-7)^2 + 1^2}$$
$$|GH| = \sqrt{50}$$
$$|GH| = 5\sqrt{2}$$

Length of a Line Segment

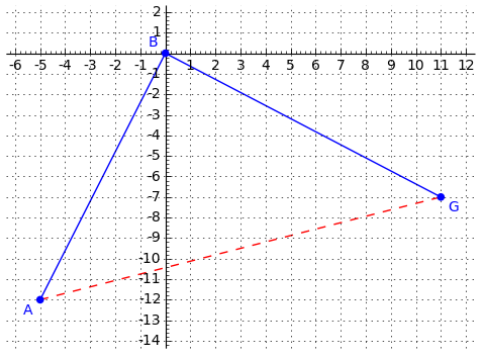
Example

Alphaville is 12 km south and 5 km west of Betatown. Gammaburg is 7 km south and 11 km east of Betatown. If a helicopter flies directly from Alphaville to Gammaburg, how far does it fly?

Let Betatown be at $B(0, 0)$ on the Cartesian plane.

If North and East are positive, while South and West are negative, then Alphaville and Gammaburg will have coordinates $A(-5, -12)$ and $G(11, -7)$ respectively.

Length of a Line Segment



Length of a Line Segment

Use the formula to calculate the distance between the two cities.

$$|AG| = \sqrt{(11-(-5))^2+(-7-(-12))^2}$$
$$|AG| = \sqrt{16^2+5^2}$$
$$|AG| = \sqrt{281}$$
$$|AG| \approx 16.76$$

The helicopter flies approximately 16.76 km.

Questions?