MPM2D: Principles of Mathematics

Factoring Polynomials

Part 5: Factoring Special Cases

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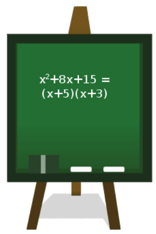

Multiplying Binomials

Recap

Expand and simplify $(4x+3)(4x-3)$.

Recall that this produces a difference of squares, where the middle term disappears.

$$\begin{aligned}(4x+3)(4x-3) &= 16x^2 - 12x + 12x - 9 \\ &= 16x^2 - 9\end{aligned}$$

Factoring Differences of Squares

Recall the general expansion of $(ax+c)(ax-c)$.

$$\begin{aligned}(ax+c)(ax-c) &= ax \cdot ax - ax \cdot c + ax \cdot c - c \cdot c \\ &= (ax)^2 - c^2\end{aligned}$$

To work backward and obtain the values of ax and c in the form $(ax+c)(ax-c)$, we can simply take the square roots of (ax^2) and c^2.

While decomposition works (using a middle coefficient of 0), it takes longer and is not recommended.

Factoring Differences of Squares

Example

Factor $x^2 - 4$.

This is a difference of squares, so we can take the square roots of x^2 and 4.

$\sqrt{x^2} = x$ and $\sqrt{4} = 2$.

Therefore, $x^2 - 4 = (x+2)(x-2)$.

Factoring Differences of Squares

Example

Factor $400x^2 - 81$.

This is a difference of squares, so we can take the square roots of $400x^2$ and 81.

$\sqrt{400x^2} = \sqrt{400}\sqrt{x^2} = 20x$ and $\sqrt{81} = 9$.

Therefore, $400x^2 - 81 = (20x+9)(20x-9)$.

Factoring Differences of Squares

Example

Factor $36x^2 - 49y^2$.

This is a difference of squares, so we can take the square roots of $36x^2$ and $49y^2$.

$\sqrt{36x^2} = \sqrt{36}\sqrt{x^2} = 6x$ and $\sqrt{49y^2} = \sqrt{49}\sqrt{y^2} = 7y$.

Therefore, $36x^2 - 49y^2 = (6x+7y)(6x-7y)$.

Factoring Differences of Squares

Example

Factor $4x^2 + 25$.

While $\sqrt{4x^2} = 2x$ and $\sqrt{25} = 5$, this is a *sum* of squares, not a difference.

If the factors have the form $(ax + c)(ax - c)$, the result will be a difference of squares.

If the factors have the form $(ax + c)(ax + c) = (ax + c)^2$, or $(ax - c)(ax - c) = (ax - c)^2$, then the resulting expansion would be a perfect square with a middle term.

In general, it is not possible to factor a sum of squares, so the expression is not factorable.

Factoring Perfect Squares

Recall the general expansion of $(ax + c)^2$.

$$\begin{aligned}(ax + c)^2 &= (ax + c)(ax + c)\\ &= (ax)^2 + (2ac)x + c^2\end{aligned}$$

Similarly, $(ax - c)^2 = (ax)^2 - (2ac)x + c^2$.

The coefficient of the middle term is twice the product of a and c, and its sign is the same as the one in the binomial factor.

This means that if we can recognize a trinomial as a perfect square, there is a quick method for factoring it.

Factoring Perfect Squares

Example

Factor $x^2 + 6x + 9$.

Let $a = \sqrt{1} = 1$ and $c = \sqrt{9} = 3$.

Since $2ac = 2 \times 1 \times 3 = 6$, $x^2 + 6x + 9$ is a perfect square.

Therefore, $x^2 + 6x + 9 = (x + 3)^2$. Note that the sign is positive since the coefficient of the middle term in the standard form is 6.

Factoring Perfect Squares

Example

Factor $4x^2 - 20xy + 25y^2$

Let $a = \sqrt{4} = 2$ and $c = \sqrt{25} = 5$.

Since $2ac = 2 \times 2 \times 5 = 20$, $4x^2 - 20xy + 25y^2$ is a perfect square.

Therefore, $4x^2 - 20xy + 25y^2 = (2x - 5y)^2$. Note that the sign is negative since the coefficient of the middle term in the standard form is -20.

Factoring Perfect Squares

Example

Factor $729x^2 - 2052x + 1444$.

Using decomposition with this expression would be difficult, since the numbers are so large.

However, let $a = \sqrt{729} = 27$ and $c = \sqrt{1444} = 38$.

Since $2ac = 2 \times 27 \times 38 = 2052$, $729x^2 - 2052x + 1444$ is a perfect square.

Therefore, $729x^2 - 2052x + 1444 = (27x - 38)^2$.

Questions?