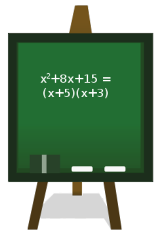
MPM2D: Principles of Mathematics

Factoring Polynomials

Part 3: Factoring Simple Trinomials

J. Garvin



Multiplying Binomials

Recap

Expand and simplify $(x-6)(x+3)$.

$$\begin{aligned}(x-6)(x+3) &= x^2+3x-6x-18\\ &= x^2-3x-18\end{aligned}$$

Factoring Simple Trinomials

Recall that a quadratic expression has the form ax^2+bx+c for some real numbers a, b and c ($a \neq 0$).

If $a=1$, then the expression will have the form x^2+bx+c. This is often referred to as a *simple trinomial*.

Simple trinomials usually result from multiplying two binomials, each of the form $x+k$.

For example, $(x+4)(x-7)=x^2-3x-28$.

While it is straightforward to expand the product two binomials into a simple trinomial, how can we reverse the process and factor a simple trinomial into the product of two binomials?

Factoring Simple Trinomials

To develop a method for factoring simple trinomials, consider the expression $(x+p)(x+q)$, where p and q are any real numbers.

Use the Distributive Law to expand.

$$(x+p)(x+q)=x^2+px+qx+pq$$

The two middle terms share a common factor of x.

$$(x+p)(x+q)=x^2+(p+q)x+pq$$

There are two things to note:

- The constant term is the product pq.
- The coefficient of the middle term is the sum $p+q$.

Factoring Simple Trinomials

To rewrite a simple trinomial of the form x^2+bx+c as $(x+p)(x+q)$, we must determine two numbers p and q such that $c=pq$ and $b=p+q$.

Begin by determining all factors of c, and see what combination of factors (if any) have a sum of b.

Consider the simple trinomial x^2+5x+6.

The factors of the constant term, 6, are 1, 2, 3 and 6.

Note that while $1\times 6=6$, $1+6=7\neq b$.

However, $2\times 3=6$ and $2+3=5=b$. Therefore, $p=2$ and $q=3$ (order does not matter).

This means that $x^2+5x+6=(x+2)(x+3)$.

Factoring Simple Trinomials

Example

Factor $x^2+16x+15$.

The factors of the constant term, 15, are 1, 3, 5 and 15.

Of these factors, $1\times 15=15$ and $1+15=16$.

If $p=1$ and $q=15$, then $x^2+16x+15=(x+1)(x+15)$.

We cannot use $p=3$ and $q=5$, since $3+5=8\neq 16$.

Note that there will always be one unique combination of factors (if any) that work.

Factoring Simple Trinomials

Example

Factor $x^2 + 11x + 24$.

There are many factors of 24: 1, 2, 3, 4, 6, 8, 12 and 24.

Of these factors, only $3 \times 8 = 24$ and $3 + 8 = 11$.

If $p = 3$ and $q = 8$, then $x^2 + 11x + 24 = (x + 3)(x + 8)$.

In all of the previous examples, we looked for factors that were always positive, but it is also possible for factors to be negative.

The signs of the constant and middle terms can help us determine which factors are the correct ones.

Factoring Simple Trinomials

Example

Factor $x^2 - 7x + 10$.

Since c is positive, p and q must both be positive or both be negative.

The middle term, b, is negative. Since the sum of two negative numbers is negative, p and q must both be negative.

The factors of 10 are 1, 2, 5 and 10.

Since $(-2)(-5) = 10$ and $-2 + (-5) = -7$, $p = -2$ and $q = -5$.

Therefore, $x^2 - 7x + 10 = (x - 2)(x - 5)$.

Factoring Simple Trinomials

Example

Factor $x^2 + x - 12$.

Remember that the middle term has a coefficient of 1.

Since c is negative, p will be positive while q will be negative.

The middle term, b, is positive. Therefore, $|p| > |q|$. That is, the magnitude of the positive factor is greater than the magnitude of the negative factor.

The factors of 12 are 1, 2, 3, 4, 6 and 12.

Since $4(-3) = -12$ and $4 + (-3) = 1$, $p = 4$ and $q = -3$.

Therefore, $x^2 + x - 12 = (x + 4)(x - 3)$.

Factoring Simple Trinomials

Example

Factor $x^2 - 7x - 18$.

Since c is negative, p will be positive while q will be negative.

The middle term, b, is negative. Therefore, $|p| < |q|$. That is, the magnitude of the positive factor is less than the magnitude of the negative factor.

The factors of 18 are 1, 2, 3, 6, 9 and 18.

Since $2(-9) = -18$ and $2 + (-9) = -7$, $p = 2$ and $q = -9$.

Therefore, $x^2 - 7x - 18 = (x + 2)(x - 9)$.

Factoring Simple Trinomials

Example

Factor $x^2 + 5x + 2$.

Since b and c are both positive, both p and q must be positive as well.

The only factors of 2, however, are 1 and 2, and $1 + 2 = 3 \neq 5$.

Since there are no values of p and q with a product of 2 and a sum of 5, the trinomial is not factorable.

Questions?