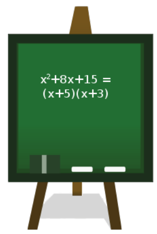
MPM2D: Principles of Mathematics

Factoring Polynomials

Part 4: Factoring Complex Trinomials

J. Garvin



Multiplying Binomials

Recap

Expand and simplify $(2x+5)(3x+4)$.

$$\begin{aligned}(2x+5)(3x+4) &= 6x^2+8x+15x+20\\ &= 6x^2+23x+20\end{aligned}$$

The resulting expression is called a *complex trinomial*, since the leading coefficient is not 1.

Multiplying Binomials

Now consider the expansion of the expression $(ex+f)(gx+h)$, where e, f, g and h are any integers.

$$\begin{aligned}(ex+f)(gx+h) &= ex\cdot gx+ex\cdot h+f\cdot gx+f\cdot h\\ &= egx^2+ehx+fgx+fh\\ &= egx^2+(eh+fg)x+fh\end{aligned}$$

Since the standard form of a quadratic expression is ax^2+bx+c, the values of a, b and c are related to e, f, g and h as follows:

$$a=eg \qquad b=eh+fg \qquad c=fh$$

Multiplying Binomials

Like simple trinomials, the coefficient of the middle term, b, is the sum of two values, in this case eh and fg.

Note that $ac = eg\cdot fh = eh\cdot fg$ (since the order of multiplication does not matter).

These are the two values that sum to b.

Therefore, for any factorable trinomial ax^2+bx+c, there are two values with a sum of b and a product of ac.

In the case of a simple trinomial ($a=1$), the product $ac=(1)c=c$ as before.

Factoring Complex Trinomials

To work backward and factor a complex trinomial, we must determine the two values, p and q, with a sum of b and a product of ac.

To make things work, we *decompose* the middle term into separate two terms with coefficients p and q.

This process is often referred to as *decomposition*.

The resulting expression will look like this:

$$ax^2+px+qx+c$$

Since ac, p and q all share the same factors, it will be possible to use grouping to find common factors.

Some examples should make this clearer.

Factoring Complex Trinomials

Example

Factor $2x^2+13x+15$.

We must determine two numbers, p and q, that have a product of $2\times 15=30$ and a sum of 13.

Since the product and sum are both positive, both p and q will be positive.

Factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30.

The two numbers that meet the requirements are 3 and 10.

Factoring Complex Trinomials

Decompose the middle term into two terms with p and q as coefficients.

$$2x^2 + 13x + 15 = 2x^2 + 3x + 10x + 15$$

Factor by grouping.

$$\begin{aligned} 2x^2 + 3x + 10x + 15 &= x(2x+3) + 5(2x+3) \\ &= (2x+3)(x+5) \end{aligned}$$

Therefore, $2x^2 + 13x + 15 = (2x+3)(x+5)$.

Factoring Complex Trinomials

Example

Factor $6x^2 + 19x - 7$.

We must determine two numbers, p and q, that have a product of $6(-7) = -42$ and a sum of 19.

Since the product is negative and the sum is positive, p will be positive and $|p| > |q|$.

Factors of 42 are 1, 2, 3, 6, 7, 14, 21 and 42.

The two numbers that meet the requirements are 21 and -2.

Factoring Complex Trinomials

Decompose the middle term into two terms with p and q as coefficients.

$$6x^2 + 19x - 7 = 6x^2 + 21x - 2x - 7$$

Factor by grouping.

$$\begin{aligned} 6x^2 + 21x - 2x - 7 &= 3x(2x+7) - 1(2x+7) \\ &= (2x+7)(3x-1) \end{aligned}$$

Therefore, $6x^2 + 19x - 7 = (2x+7)(3x-1)$.

Factoring Complex Trinomials

Example

Factor $25x^2 - 20x + 4$.

We must determine two numbers, p and q, that have a product of $25 \times 4 = 100$ and a sum of -20.

Since the product is positive and the sum is negative, both p and q will be negative.

Factors of 100 are 1, 2, 4, 5, 10, 20, 25, 50 and 100.

In this case, the two numbers that meet the requirements have the same value: -10 and -10.

Factoring Complex Trinomials

Decompose the middle term into two terms with p and q as coefficients.

$$25x^2 - 20x + 4 = 25x^2 - 10x - 10x + 4$$

Factor by grouping.

$$\begin{aligned} 25x^2 - 10x - 10x + 4 &= 5x(5x-2) - 2(5x-2) \\ &= (5x-2)(5x-2) \end{aligned}$$

Therefore, $25x^2 - 20x + 4 = (5x-2)(5x-2)$, or $(5x-2)^2$.

While decomposition works for perfect squares like the one above, we will look at a more efficient method in the next lesson.

Questions?