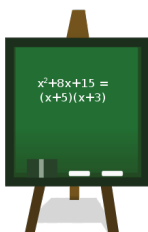


Factoring Polynomials

Part 2: Factoring By Grouping

J. Garvin



Slide 1/12

Common Factoring

Recap

Factor $12x^4 + 14x^3 - 20x^2$.

The GCF is $2x^2$, since 12, 14 and -20 are all divisible by 2 but no larger number, and the highest exponent of x is 2.

$$\begin{aligned} 12x^4 + 14x^3 - 20x^2 &= 2x^2 \left(\frac{12x^4}{2x^2} + \frac{14x^3}{2x^2} - \frac{20x^2}{2x^2} \right) \\ &= 2x^2(6x^2 + 7x - 10) \end{aligned}$$

J. Garvin — Factoring Polynomials
Slide 2/12

Expressions with Common Binomial Factors

In the recap, we were looking for the greatest common factor in all terms of the polynomial expression.

In general, if a common factor k is present in all terms, then the polynomial will be rewritten as $k(ax^b + cx^d + ex^f + \dots)$.

Expanding this gives $ax^b(k) + cx^d(k) + ex^f(k) + \dots$

This is true even if k is an *expression*, rather than a constant or variable.

For example, $(x + 1)(x^2 + 5) = x^2(x + 1) + 5(x + 1)$.

In this case, $(x + 1)$ is a binomial that is common to both terms in the expanded expression.

J. Garvin — Factoring Polynomials
Slide 3/12

Expressions with Common Binomial Factors

Example

Factor $3x(x - 5) + 2(x - 5)$.

Let $k = x - 5$. Then the expression becomes $3xk + 2k$.

Common factoring out k gives $3xk + 2k = k(3x + 2)$.

Substitute $k = x - 5$ to obtain $k(3x + 2) = (x - 5)(3x + 2)$.

Therefore, $3x(x - 5) + 2(x - 5) = (x - 5)(3x + 2)$.

J. Garvin — Factoring Polynomials
Slide 4/12

Expressions with Common Binomial Factors

Example

Factor $5x^2(2x + 7) - (2x + 7)$.

Let $k = 2x + 7$. Then the expression becomes $5x^2k - k$.

Common factoring out k gives $5x^2k - k = k(5x^2 - 1)$. Remember the 1 as a placeholder.

Substiting $k = 2x + 7$, $k(5x^2 - 1) = (2x + 7)(5x^2 - 1)$.

Therefore, $5x^2(2x + 7) - (2x + 7) = (2x + 7)(5x^2 - 1)$.

J. Garvin — Factoring Polynomials
Slide 5/12

Factoring By Grouping

Now consider the cubic expression $x^3 + 2x^2 + 8x + 16$.

Normally, factoring a cubic expression is a task saved for higher-level math courses, as it involves more complex concepts.

In this case, we can “group” the first two terms together, and the last two terms together, and find a common factor for each group.

$$\begin{aligned} x^3 + 2x^2 + 8x + 16 &= [x^3 + 2x^2] + [8x + 16] \\ &= x^2(x + 2) + 8(x + 2) \end{aligned}$$

Notice that there are now two terms in the expression, each with a common factor of $(x + 2)$.

J. Garvin — Factoring Polynomials
Slide 6/12

Factoring By Grouping

We can now common factor the expression like we did before.

$$x^2(x+2) + 8(x+2) = (x+2)(x^2+8)$$

Since we factor the expression by grouping together terms, this method is generally called *factoring by grouping*.

In this course, polynomial expressions that contain four terms are good candidates for factoring by grouping.

Factoring By Grouping

Example

Factor $3x^3 - 15x^2 - 4x + 20$.

Group the first two, and the last two, terms and look for a common factor. Watch the signs!

$$\begin{aligned} 3x^3 - 15x^2 - 4x + 20 &= [3x^3 - 15x^2] - [4x - 20] \\ &= 3x^2(x - 5) - 4(x - 5) \\ &= (x - 5)(3x^2 - 4) \end{aligned}$$

Factoring By Grouping

Example

Factor $x^3 - 2x^2 + 6x + 9$.

Again, group adjacent terms and find a common factor.

$$\begin{aligned} x^3 - 2x^2 + 6x + 9 &= [x^3 - 2x^2] + [6x + 9] \\ &= x^2(x - 2) + 3(2x + 3) \end{aligned}$$

Since we obtain different expressions inside of the brackets, there is no common factor.

Therefore, we are unable to factor this expression by grouping.

Factoring By Grouping

Example

Factor $xy - 4y - 3x + 12$.

The first two terms contain y , indicating it may be possible to factor by grouping.

$$\begin{aligned} xy - 4y - 3x + 12 &= [xy - 4y] - [3x - 12] \\ &= y(x - 4) - 3(x - 4) \\ &= (x - 4)(y - 3) \end{aligned}$$

Factoring By Grouping

Example

Factor $2a^2 + 21b - 6ab - 7a$.

We cannot group the terms as they currently are, since the GCF of $2a^2$ and $21b$ is 1.

It is possible, however, to move some of the terms such that the expression can be factored.

$$\begin{aligned} 2a^2 + 21b - 6ab - 7a &= 2a^2 - 6ab - 7a + 21b \\ &= 2a(a - 3b) - 7(a - 3b) \\ &= (a - 3b)(2a - 7) \end{aligned}$$

Questions?

