MPM2D: Principles of Mathematics

Quadratic Relations

Quadratics In Factored Form

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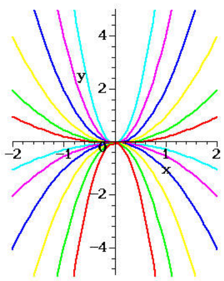


Distributive Law

Recall

Expand and simplify the expression $3(x+4)(x-5)$.

$$\begin{aligned} 3(x+4)(x-5) &= (3x+12)(x-5) \\ &= 3x^2-15x+12x-60 \\ &= 3x^2-3x-60 \end{aligned}$$

This could have also been solved by multiplying the two binomials first, then multiplying the terms of the resulting trinomial by 3.

Quadratic Relations

Consider the quadratic relation $y=2(x-1)(x-5)$.

Expanding the equation using the distributive law gives the equivalent equation $y=2x^2-12x+10$.

Constructing a table of values for either expression yields the same values.

$y=2(x-1)(x-5)$

x	y
0	$2(0-1)(0-5)=10$
1	$2(1-1)(1-5)=0$
2	$2(2-1)(2-5)=-6$
3	$2(3-1)(3-5)=-8$
4	$2(4-1)(4-5)=-6$
5	$2(5-1)(5-5)=0$
6	$2(6-1)(6-5)=10$

$y=2x^2-12x+10$

x	y
0	$2(0)^2-12(0)+10=10$
1	$2(1)^2-12(1)+10=0$
2	$2(2)^2-12(2)+10=-6$
3	$2(3)^2-12(3)+10=-8$
4	$2(4)^2-12(4)+10=-6$
5	$2(5)^2-12(5)+10=0$
6	$2(6)^2-12(6)+10=10$

Quadratic Relations

A quadratic relation written as the product of two binomials is said to be in *factored form*.

Note that in its table of values, $y=2(x-1)(x-5)$ has x-intercepts at $(1,0)$ and $(5,0)$. That is, the signs of the factors appear oppsite.

Also note that the y-intercept is at $(0,10)$, and that $(2)(1)(5)=10$.

Factored Form of a Quadratic

A quadratic relation in factored form, $y=a(x-r)(x-s)$, has x-intercepts of r and s. It opens upward if $a>0$, and downward if $a<0$. Its y-intercept is $a \cdot r \cdot s$.

Quadratic Relations

Example

State the intercepts for the relation $y=2(x-3)(x-8)$.

Since $r=3$ and $s=8$, the relation has x-intercepts at $(3,0)$ and $(8,0)$.

Since $2(3)(8)=48$, the y-intercept is at $(0,48)$.

Example

State the intercepts for the relation $y=-3x(x+7)$.

The relation can be rewritten $y=-3(x-0)(x-(-7))$, with x-intercepts at $(0,0)$ and $(-7,0)$.

In this case, $(0,0)$ is also the y-intercept.

Quadratic Relations

Recall that a parabola's vertex lies on its axis of symmetry.

The axis is midway between any two points that have the same y-coordinate, such as the x-intercepts.

The x-coordinate of the vertex, then, is the average of the x-coordinates of the x-intercepts.

The y-coordinate of the vertex can be calculated using the equation of the relation.

Factored Form of a Quadratic

The x-coordinate of the vertex, (h,k), of a parabola described by the relation $y=a(x-r)(x-s)$, is at $h=\frac{r+s}{2}$. The y-coordinate is at $k=a(h-r)(h-s)$.

Quadratic Relations

Example

Determine the coordinates of the vertex of the parabola described by $y = 3(x+2)(x-4)$.

The x-intercepts are at $(-2, 0)$ and $(4, 0)$.

The x-coordinate of the vertex is the average of -2 and 4, or $\frac{-2+4}{2} = 1$.

To determine the y-coordinate of the vertex, substitute $x = 1$ into the equation.

$$\begin{aligned} y &= 3(1+2)(1-4) \\ &= -27 \end{aligned}$$

Therefore, the vertex is at $(1, -27)$.

Quadratic Relations

Example

Graph the quadratic relation $y = -(x-3)(x+5)$.

The x-intercepts are at $(3, 0)$ and $(-5, 0)$, and the y-intercept is at $(0, 15)$.

The x-coordinate of the vertex is $\frac{3-5}{2} = -1$.

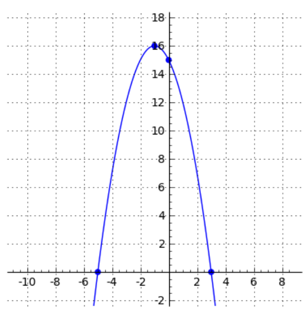
Substitute $x = -1$ into the equation.

$$\begin{aligned} y &= -(-1-3)(-1+5) \\ &= 16 \end{aligned}$$

Therefore, the vertex is at $(-1, 16)$.

These four points are probably sufficient to sketch the parabola, but additional points can be generated using the step pattern $-1, -3, -5, \ldots$.

Quadratic Relations



Quadratic Relations

Example

Determine an equation for a quadratic relation with x-intercepts at $(7, 0)$ and $(-1, 0)$, if it passes through $(2, 30)$.

Use $r = 7$, $s = -1$, $x = 2$ and $y = 30$ in factored form, then solve for a.

$$\begin{aligned} y &= a(x-r)(x-s) \\ 30 &= a(2-7)(2+1) \\ 30 &= -15a \\ a &= -2 \end{aligned}$$

An equation that meets the criteria is $y = -2(x-7)(x+1)$.

Questions?