MPM2D: Principles of Mathematics

Exponent Laws

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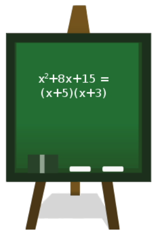

Exponent Laws

Consider the expression $x^2 \cdot x^3$.

Using the definition of exponentiation, $x^2 \cdot x^3$ can be expressed as $(x \cdot x)(x \cdot x \cdot x) = x \cdot x \cdot x \cdot x \cdot x = x^5$.

More generally, $x^a \cdot x^b = \underbrace{(x \cdot x \cdot \ldots \cdot x)}_{\text{a times}} \cdot \underbrace{(x \cdot x \cdot \ldots \cdot x)}_{\text{b times}}$

$= \underbrace{x \cdot x \cdot \ldots \cdot x}_{\text{a+b times}} = x^{a+b}$.

Product of Like Powers Law

For any real, non-zero values a, b and x, $x^a \cdot x^b = x^{a+b}$.

If the bases are not the same, this rule does not apply. The expression $2^4 \cdot 3^2$ cannot be simplified further.

Exponent Laws

Next, consider the expression $\frac{x^3}{x^2}$.

Rewriting, $\frac{x^3}{x^2}$ can be expressed as $\frac{x \cdot x \cdot x}{x \cdot x} = x$.

More generally, $\frac{x^a}{x^b} = \frac{\overbrace{(x \cdot x \cdot \ldots \cdot x)}^{\text{a times}}}{\underbrace{(x \cdot x \cdot \ldots \cdot x)}_{\text{b times}}} = \underbrace{x \cdot x \cdot \ldots \cdot x}_{\text{a-b times}} = x^{a-b}$.

Quotient of Like Powers Law

For any real, non-zero values a, b and x, $\frac{x^a}{x^b} = x^{a-b}$.

Like the earlier Product Law, the bases must be the same.

Exponent Laws

Now, consider $\left(x^3\right)^2$.

Rewriting, $\left(x^3\right)^2$ becomes $(x \cdot x \cdot x) \cdot (x \cdot x \cdot x) = x^6$.

In general, $\left(x^a\right)^b =$

$\underbrace{\underbrace{(x \cdot x \cdot \ldots \cdot x)}_{\text{a times}} \cdot \underbrace{(x \cdot x \cdot \ldots \cdot x)}_{\text{a times}} \cdot \ldots \cdot \underbrace{(x \cdot x \cdot \ldots \cdot x)}_{\text{a times}}}_{\text{b times}} = x^{ab}$.

Power of a Power Law

For any real, non-zero values a, b and x, $\left(x^a\right)^b = x^{ab}$.

Exponent Laws

Consider $(xy)^2$ next.

In its longer form, $(xy)^2 = (xy)(xy) = (x \cdot x)(y \cdot y) = x^2y^2$.

In general, $(xy)^a = \underbrace{(xy) \cdot (xy) \cdot \ldots \cdot (xy)}_{\text{a times}} =$

$\underbrace{(x \cdot x \cdot \ldots \cdot x)}_{\text{a times}} \cdot \underbrace{(y \cdot y \cdot \ldots \cdot y)}_{\text{a times}} = x^a y^a$.

Power of a Product Law

For any real, non-zero values a, x and y, $(xy)^a = x^a y^a$.

Exponent Laws

Like the power of a product, the power of a quotient can be similarly defined.

For instance, $\left(\frac{x}{y}\right)^2 = \left(\frac{x}{y}\right) \cdot \left(\frac{x}{y}\right) = \frac{x^2}{y^2}$.

In general, $\left(\frac{x}{y}\right)^a = \underbrace{\left(\frac{x}{y}\right) \cdot \left(\frac{x}{y}\right) \cdot \ldots \cdot \left(\frac{x}{y}\right)}_{\text{a times}} = \frac{x^a}{y^a}$

Power of a Quotient Law

For any real, non-zero values a, x and y, $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$.

Exponent Laws

Example

Simply the expressions $x^4 \cdot x^7$, $\frac{z^8}{z^6}$, $\left(k^3\right)^5$, $(2p)^3$ and $\left(\frac{x}{2}\right)^5$.

$x^4 \cdot x^7 = x^{4+7} = x^{11}$.

$\frac{z^8}{z^6} = z^{8-6} = z^2$.

$\left(k^3\right)^5 = k^{3\times 5} = k^{15}$.

$(2p)^3 = 2^3p^3 = 8p^3$.

$\left(\frac{x}{2}\right)^5 = \frac{x^5}{2^5} = \frac{x^5}{32}$.

Exponent Laws

What about the expression x^0?

According to the Quotient Law, $\frac{x^a}{x^a} = x^{a-a} = x^0$.

At the same time, $\frac{k}{k} = 1$, as long as $k \neq 0$.

If $k = x^a$, then $\frac{k}{k} = \frac{x^a}{x^a} = x^0 = 1$.

Zero Exponent Law

For any real, non-zero value of x, $x^0 = 1$.

Exponent Laws

What does a negative exponent, like x^{-2}, mean?

Since $\frac{x}{x^3} = x^{1-3} = x^{-2}$, and since $\frac{x}{x^3} = \frac{1}{x^2}$, then $x^{-2} = \frac{1}{x^2}$.

In general, $x^a \cdot x^{-a} = x^{a+(-a)} = x^0 = 1$, assuming $x \neq 0$.

Therefore, $x^a \cdot x^{-a} = 1$, which can be rearranged to $x^{-a} = \frac{1}{x^a}$.

Negative Exponent Law

For any real, non-zero value of x and any real, positive value of a, $x^{-a} = \frac{1}{x^a}$.

Exponent Laws

Example

Evaluate $1\,234\,567^0$.

Since the base is non-zero, $1\,234\,567^0 = 1$.

Example

Express x^{-4} using positive exponents.

$x^{-4} = \frac{1}{x^4}$. Again, x cannot equal zero.

Exponent Laws

Sometimes it is necessary to combine two or more exponent laws to simplify an expression.

Example

Simplify $\frac{x^5y^3}{x^2y^7}$, using positive exponents.

$$\begin{aligned}\frac{x^5y^3}{x^2y^7} &= x^{5-2}y^{3-7}\\ &= x^3y^{-4}\\ &= \frac{x^3}{y^4}\end{aligned}$$

Exponent Laws

Example

Simplify $(5p^{-3}q)^{-2}$, using positive exponents.

$$\begin{aligned}(5p^{-3}q)^{-2} &= 5^{-2}p^{(-3)(-2)}q^{-2}\\ &= \frac{1}{5^2}\cdot p^6 \cdot \frac{1}{q^2}\\ &= \frac{p^6}{25q^2}\end{aligned}$$

Scientific Notation

Scientific notation is a system, used in many sciences, that expresses numbers using powers of 10.

For example, the number 352 can be expressed as 3.52×10^2, since $3.52 \times 10^2 = 3.52 \times 100 = 352$.

It is often used as a shorthand notation for very small or very large numbers.

For instance, 3 800 000 000 000 (3 trillion, 800 billion) can be expressed more simply as 3.8×10^{12}.

By convention, scientific notation expresses all numbers with one digit before the decimal point – that is, 4.3×10^3 rather than 43×10^2.

Positive exponents indicate the decimal point has been shifted left, while negative exponents indicate a right shift.

Scientific Notation

Example

Express 75 328 143 using scientific notation, to two decimal places.

Shifting the decimal 7 places to the left, and rounding down, $75\,328\,143 = 7.53 \times 10^7$.

Example

Express 0.000 031 874 using scientific notation, to two decimal places.

Shifting the decimal 5 places to the right, and rounding up, $0.000\,031\,874 = 3.19 \times 10^{-5}$.

Questions?