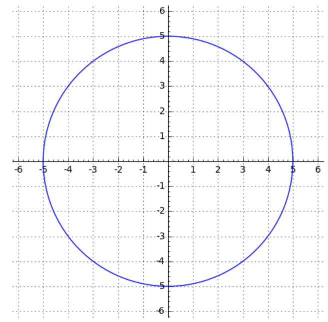
MPM2D: Principles of Mathematics

Equations of Circles

J. Garvin

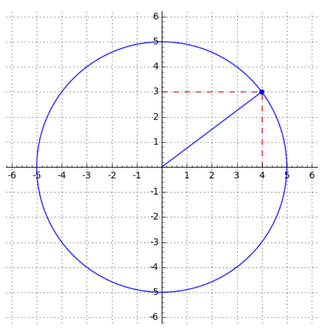
Equations of Circles

Consider a circle centred at the origin with a radius of 5 units, as shown below.



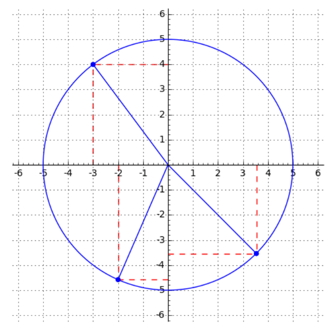
Equations of Circles

By constructing a radius from $(0, 0)$ to $(4, 3)$, we form a right triangle whose arms have lengths 4 and 3 units.



Equations of Circles

A right triangle is formed regardless of where the radius is constructed.



Equations of Circles

In all cases, the hypotenuse of any right triangle formed is a radius of the circle, r.

If the horizontal arm of a right triangle formed has a length of x units, and the vertical arm has a length of y units, then the length of the hypotenuse can be calculated using the Pythagorean Theorem.

This gives us an equation for a circle, centred at the origin.

Equation of a Circle Centred at the Origin

A circle with radius r, centred at the origin, has equation $x^2 + y^2 = r^2$.

Equations of Circles

Example

Determine the equation of a circle, centred at the origin, with a radius of 3 units.

Since $r = 3$, $r^2 = 9$, so the equation is $x^2 + y^2 = 9$.

Example

A circle has equation $x^2 + y^2 = 50$. Determine the length of its radius.

Since $r^2 = 50$, the radius is $r = \sqrt{50} = 5\sqrt{2}$ units long.

Equations of Circles

Example

Determine the equation, and length of the radius, of a circle centred at the origin that passes through $P(-2, 3)$.

Substitute $x = -2$ and $y = 3$ into the equation, to determine the value of r^2.

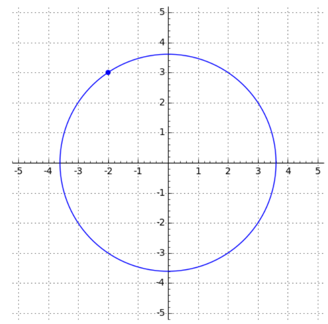
$$\begin{aligned}(-2)^2 + 3^2 &= 4 + 9 \\ &= 13\end{aligned}$$

Therefore, the equation of the circle is $x^2 + y^2 = 13$.

The radius is $\sqrt{13} \approx 3.6$ units.

Equations of Circles

Graphing the circle and point confirms the calculations.



Equations of Circles

Example

Determine whether $P(6, -8)$ is on, inside, or outside of the circle with radius 10.

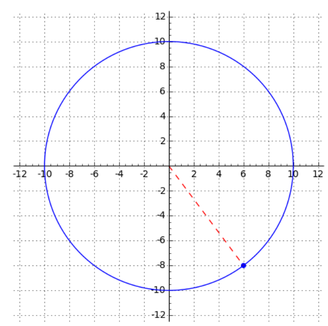
Substitute $x = 6$ and $y = -8$ into the equation.

$$\begin{aligned}6^2 + (-8)^2 &= r^2 \\ r^2 &= 100 \\ r &= 10\end{aligned}$$

Since the equation is satisfied, P is on the circle.

Equations of Circles

Graphing the circle and point confirms the calculations.



Equations of Circles

Example

Determine whether $P(-4, 5)$ is on, inside, or outside of the circle with equation $x^2 + y^2 = 36$.

Substitute $x = -4$ and $y = 5$ into the equation.

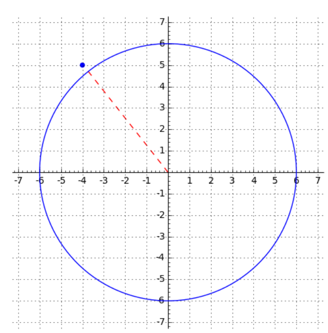
$$\begin{aligned}(-4)^2 + 5^2 &= r^2 \\ r^2 &= 41\end{aligned}$$

Since $41 > 36$, $\sqrt{41} > \sqrt{36}$. This means that the calculated hypotenuse has a length greater than the radius of the circle.

Therefore, Q is outside of the circle.

Equations of Circles

Graphing the circle, point Q is slightly outside of the circle.



Questions?