MPM2D: Principles of Mathematics

Circumcentre of a Triangle

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Equation of a Right Bisector

Recap

Determine the equation of the right bisector of the line segment from $A(-4,-7)$ to $B(10,1)$.

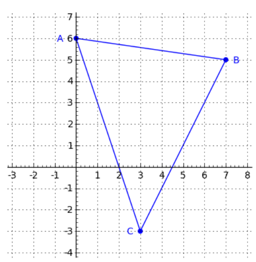
$$M_{AB} = \left(\frac{-4+10}{2}, \frac{-7+1}{2}\right) = (3,-3) \qquad m_{AB} = \frac{-7-1}{-4-10} = \frac{4}{7}$$

$$m_{RB} = -\tfrac{7}{4}$$
$$-3 = -\tfrac{7}{4}(3) + b$$
$$b = \tfrac{9}{4}$$

The right bisector has equation $y = -\frac{7}{4}x + \frac{9}{4}$.

Right Bisectors In a Triangle

Consider ΔABC below, with vertices at $A(0,6)$, $B(7,5)$ and $C(3,-3)$.



Right Bisectors In a Triangle

We can construct the right bisectors of AC and BC using the same process.

$$M_{AC} = \left(\frac{0+3}{2}, \frac{6-3}{2}\right) = \left(\tfrac{3}{2}, \tfrac{3}{2}\right) \qquad M_{BC} = \left(\frac{3+7}{2}, \frac{-3+5}{2}\right) = (5,1)$$

$$m_{AC} = \frac{6-(-3)}{0-3} = -3 \qquad m_{BC} = \frac{5-(-3)}{7-3} = 2$$

$$m_{RB} = \tfrac{1}{3} \qquad m_{RB} = -\tfrac{1}{2}$$

$$\tfrac{3}{2} = \tfrac{1}{3}\left(\tfrac{3}{2}\right) + b \qquad 1 = -\tfrac{1}{2}(5) + b$$

$$b = 1 \qquad b = \tfrac{7}{2}$$

$$y = \tfrac{1}{3}x + 1 \qquad y = -\tfrac{1}{2}x + \tfrac{7}{2}$$

Right Bisectors In a Triangle

Since the right bisectors have different slopes, they must intersect somewhere.

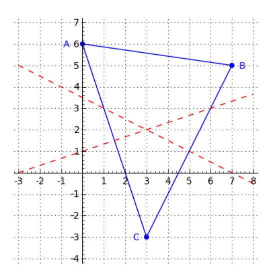
Use substitution to determine the point of intersection.

$$\tfrac{1}{3}x + 1 = -\tfrac{1}{2}x + \tfrac{7}{2}$$
$$2x + 6 = -3x + 21$$
$$5x = 15$$
$$x = 3$$
$$y = \tfrac{1}{3}(3) + 1$$
$$y = 2$$

The point of intersection is $(3,2)$.

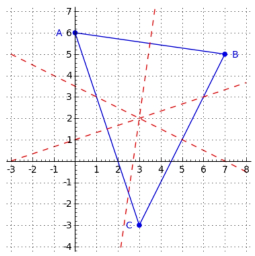
Right Bisectors In a Triangle

Graphing the two right bisectors confirms the point of intersection.



Right Bisectors In a Triangle

The right bisector of the third side also passes through the same point.



The point of intersection is called the *circumcentre*.

Circumcentre of a Triangle

Note that the distance from the circumcentre, P, to each vertex is the same.

$$|AP| = \sqrt{(3-0)^2 + (2-6)^2} = 5$$
$$|BP| = \sqrt{(3-7)^2 + (2-5)^2} = 5$$
$$|CP| = \sqrt{(3-3)^2 + (2-(-3))^2} = 5$$

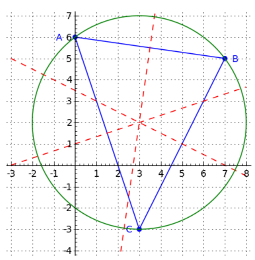
This gives us a definition for the circumcentre.

Circumcentre of a Triangle

The right bisectors of the sides of a triangle intersect at a point called the circumcentre. The circumcentre is equidistant from all three vertices of the triangle.

Circumcentre of a Triangle

Since the vertices are equidistant from the circumcentre, they form a circle *circumscribing* the triangle.



Circumcentre of a Triangle

To find the circumcentre of a triangle, follow the steps outlined on the previous slides.

1. Determine the midpoint of one side.
2. Determine the slope of that side.
3. Determine the perpendicular slope to that side.
4. Use the perpendicular slope and the midpoint to determine the equation of the right bisector of that side.
5. Repeat steps 1-4 for another side.
6. Find the point of intersection of the two right bisectors.

Sometimes there are shortcuts to make this process faster.

Circumcentre of a Triangle

Example

Determine the radius of the circle that passes through $P(-7, 2)$, $Q(11, -10)$ and $R(11, 14)$.

A circle centred at the origin cannot pass through all three of these points, since $(-7)^2 + 2^2 = 53$ and $11^2 + (-10)^2 = 221$.

By treating the three points as vertices of a triangle, we can determine the location of the circumcentre.

Once this location is known, its distance from one of the vertices can be calculated.

Circumcentre of a Triangle

The line segment connecting $Q(11, -10)$ and $R(11, 14)$ is vertical, since the x-coordinates are the same.

Therefore, the midpoint M_{QR} is at $\left(11, \frac{-10+14}{2}\right) = (11, 2)$.

Since QR is a vertical line, the right bisector of QR is a horizontal line.

As this line passes through $(11, 2)$, its equation is $y = 2$.

Always look for vertical or horizontal line segments, since equations for their right bisectors are relatively easy to determine.

Circumcentre of a Triangle

Next, find the equation of the right bisector of another side, such as PR.

$$\begin{aligned}
M_{PR} &= \left(\frac{-7+11}{2}, \frac{2+14}{2}\right) \\
&= (2,8) \\
m_{PR} &= \frac{14-2}{11+7} \\
&= \tfrac{2}{3} \\
m_{RB} &= -\tfrac{3}{2} \\
8 &= -\tfrac{3}{2}(2) + b \\
b &= 11 \\
y &= -\tfrac{3}{2}x + 11
\end{aligned}$$

Circumcentre of a Triangle

Substitute $y = 2$ into the equation to find the point of intersection.

$$\begin{aligned}
2 &= -\tfrac{3}{2}x + 11 \\
4 &= -3x + 22 \\
x &= 6
\end{aligned}$$

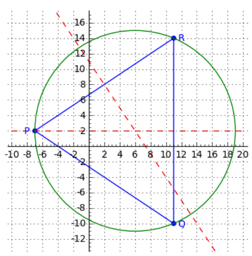
The point of intersection is $(6,2)$.

Calculate the distance from $(6,2)$ to a vertex, $(-7,2)$.

$$d = \sqrt{(-7-6)^2 + (2-2)^2} = 13$$

Therefore, the radius of the circle is 13 units.

Circumcentre of a Triangle



Questions?