MPM2D: Principles of Mathematics

Centroid of a Triangle

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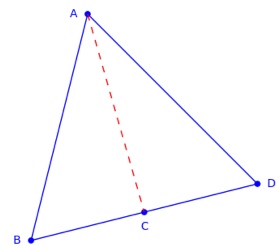
Medians

Consider ΔABD below.



Medians

The line segment AC, connecting vertex A to the midpoint of BD, is called a *median*.

A median connects a vertex to the midpoint of its opposite side.

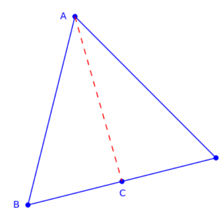
A median divides a triangle into two smaller triangles that have equal areas.

These triangles may be *congruent*, but only when the triangle is equilateral or isosceles.

Medians

Example

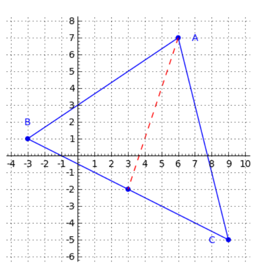
In ΔABD, $|BC| = |CD|$. If ΔABC has an area of 12 cm^2. Determine the area of ΔABD.



Since $A_{ABC} = 12$, $A_{ABD} = 2 \times 12 = 24$ cm^2.

Medians

Consider ΔABC with vertices at $A(6,7)$, $B(-3,1)$ and $C(9,-5)$ below, and the median from A.



Medians

To determine an equation for a line segment containing a median from a specific vertex, we must first determine the midpoint of the opposite side.

In the case of the median from A, we want M_{BC}.

$$M_{BC} = \left(\frac{-3+9}{2}, \frac{1+(-5)}{2}\right)$$
$$= (3,-2)$$

Now we know two points on the median: A and M_{BC}. Use these to calculate the slope of the median.

$$m_{AM} = \frac{-2-7}{3-6}$$
$$= 3$$

Medians

Once the slope is calculated, use it and either point to solve for the equation of the line.

$$7 = 3(6) + b$$
$$b = -11$$
$$y = 3x - 11$$

The line containing the median from A has equation $y = 3x - 11$.

Using M_{BC} instead of A will produce the same result.

$$-2 = 3(3) + b$$
$$b = -11$$
$$y = 3x - 11$$

Medians

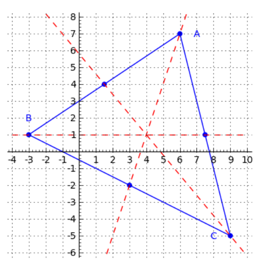
We can construct the medians from B and from C using the same process.

$$M_{AB} = \left(\frac{-3+6}{2}, \frac{1+7}{2}\right) = \left(\tfrac{3}{2}, 4\right)$$
$$m_{CM} = \frac{-5-4}{9-\frac{3}{2}} = -\tfrac{6}{5}$$
$$-5 = -\tfrac{6}{5}(9) + b$$
$$b = \tfrac{29}{5}$$
$$y = -\tfrac{6}{5}x + \tfrac{29}{5}$$

$$M_{AC} = \left(\frac{9+6}{2}, \frac{-5+7}{2}\right) = \left(\tfrac{15}{2}, 1\right)$$
$$m_{BM} = \frac{1-1}{-3-\frac{15}{2}} = 0$$
$$y = 1$$

Medians

The three medians intersect at a point called the *centroid*.



In this case, the centroid is at $(4, 1)$.

Centroid of a Triangle

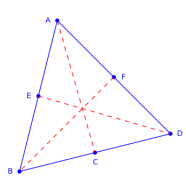
Since the medians have different slopes, we can find their point of intersection using substitution.

$$3x - 11 = -\tfrac{6}{5}x + \tfrac{29}{5}$$
$$15x - 55 = -6x + 29$$
$$21x = 84$$
$$x = 4$$
$$y = 3(4) - 11$$
$$y = 1$$

The point of intersection is $(4, 1)$

Centroid of a Triangle

When all three medians are drawn, the resulting six triangles have equal areas, "balancing" the triangle.



Centroid of a Triangle

The medians from each vertex of a triangle intersect at a point called the centroid. The centroid is the "balance point" of a triangle.

Centroid of a Triangle

Similar to the circumcentre, we can find the location of the centroid algebraically.

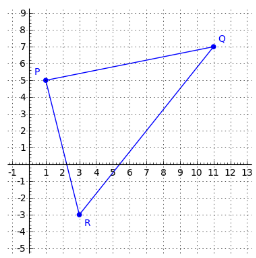
1. Determine the midpoint of a side.
2. Determine the slope from the opposite vertex to the midpoint.
3. Use the slope and a point (vertex or midpoint) to find the equation of a median.
4. Repeat steps 1-3 for another side.
5. Find the point of intersection of the two medians.

As always, shortcuts may make this process faster.

Centroid of a Triangle

Example

Determine the location of the centroid of the triangle with vertices at $P(1,5)$, $Q(11,7)$ and $R(3,-3)$.



Centroid of a Triangle

Choose any two vertices, such as P and Q, and find the equations of the medians from each vertex.

$$\begin{aligned} M_{QR} &= \left(\frac{3+11}{2}, \frac{-3+7}{2}\right) \\ &= (7,2) \\ m_{PM} &= \frac{2-5}{7-1} \\ &= -\tfrac{1}{2} \\ 5 &= -\tfrac{1}{2}(1)+b \\ b &= \tfrac{11}{2} \\ y &= -\tfrac{1}{2}x+\tfrac{11}{2} \end{aligned}$$

$$\begin{aligned} M_{PR} &= \left(\frac{1+3}{2}, \frac{5-3}{2}\right) \\ &= (2,1) \\ m_{QM} &= \frac{7-1}{11-2} \\ &= \tfrac{2}{3} \\ 7 &= \tfrac{2}{3}(11)+b \\ b &= -\tfrac{1}{3} \\ y &= \tfrac{2}{3}x-\tfrac{1}{3} \end{aligned}$$

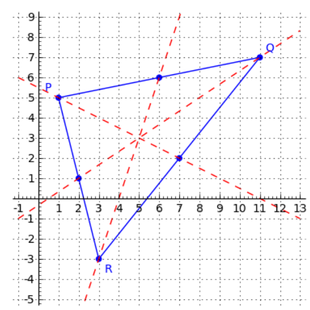
Centroid of a Triangle

Find their point of intersection using substitution.

$$\begin{aligned} -\tfrac{1}{2}x+\tfrac{11}{2} &= \tfrac{2}{3}x-\tfrac{1}{3} \\ -3x+33 &= 4x-2 \\ 7x &= 35 \\ x &= 5 \\ y &= -\tfrac{1}{2}(5)+\tfrac{11}{2} \\ y &= 3 \end{aligned}$$

The centroid is located at $(5,3)$

Centroid of a Triangle



Questions?