MPM2D: Principles of Mathematics

Right-Angled Trigonometry
Applications

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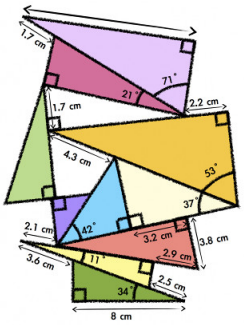
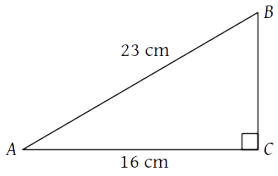


Using Trigonometric Ratios

Recap

Determine the measure of $\angle A$.



$$\cos A = \frac{16}{23}$$
$$A = \cos^{-1}\left(\frac{16}{23}\right)$$
$$A \approx 46^\circ$$

Applications of Trigonometric Ratios

Recall all of the tools available for use with right triangles:

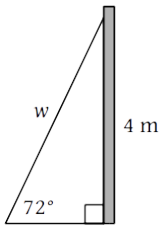
- Pythagorean Theorem (find a side, given two sides)
- trigonometric ratios (find a side, given a side and an angle)
- inverse trigonometric ratios (find an angle, given two sides)

Sometimes there is more than one way to solve a problem, but usually one method is more efficient than another.

Applications of Trigonometric Ratios

Example

A wire, attached 4 m up a telephone pole, makes an angle of 72° with the ground. How long is the wire?



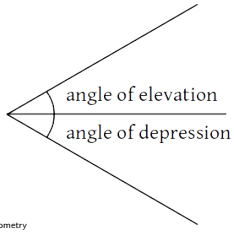
In the diagram, w is the length of the wire.

$$\sin 72^\circ = \frac{4}{w}$$
$$w = \frac{4}{\sin 72^\circ}$$
$$w \approx 4.2 \text{ m}$$

Applications of Trigonometric Ratios

Sometimes an angle is specified in relation to another object, such as "10° with the ground" or "30° to the wall".

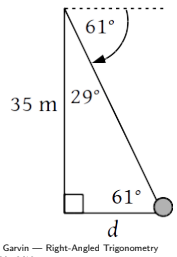
In other cases, an angle may be specified relative to a horizontal line. An angle measured upward from this line is an *angle of elevation*, while an angle measured downward is an *angle of depression*.



Applications of Trigonometric Ratios

Example

From atop of a 35 m vertical cliff, the angle of depression to a large rock below is 61°. How far from the base of the cliff is the rock?



In the diagram, d is the distance to the rock.
Note that the angle of depression falls outside of the triangle, creating a 29° angle inside.
Also note that the angle of elevation from the rock to the cliff top is 61°, due to interior alternate angles.

Applications of Trigonometric Ratios

Using either angle should produce the same answer, but will change the setup of the ratio.

Using the 29° angle...

$$\tan 29^\circ = \frac{d}{35}$$
$$d = 35 \times \tan 29^\circ$$
$$d \approx 19.4 \text{ m}$$

Using the 61° angle...

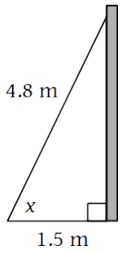
$$\tan 61^\circ = \frac{35}{d}$$
$$d = \frac{35}{\tan 61^\circ}$$
$$d \approx 19.4 \text{ m}$$

Using either method, the rock is approximately 19.4 m from the cliff.

Applications of Trigonometric Ratios

Example

The base of a 4.8 m ladder is placed 1.5 m from a wall. For safety reasons, the angle of elevation cannot exceed 67°. Is the ladder safe?



In the diagram, x is the angle of elevation.

$$\cos x = \frac{1.5}{4.8}$$
$$x = \cos^{-1}\left(\frac{1.5}{4.8}\right)$$
$$x \approx 72^\circ$$

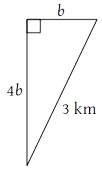
Since $72^\circ > 67^\circ$, the ladder is not safe.

Applications of Trigonometric Ratios

Example

Bert lives due east of school. Ernie lives due south of school, and is four times as far from school as Bert. If Bert and Ernie live 3 km apart, how far from school does each person live?

Let b be Bert's distance from school, and $4b$ Ernie's distance.



$$b^2 + (4b)^2 = 3^2$$
$$17b^2 = 9$$
$$b^2 = \frac{9}{17}$$
$$b = \sqrt{\frac{9}{17}}$$
$$b \approx 0.73 \text{ km}$$

Bert is about 0.73 km from school, while Ernie is $4 \times 0.73 \approx 2.92$ km.

Questions?