MPM2D: Principles of Mathematics

Applications of Similar Triangles

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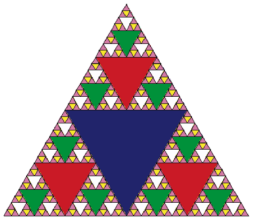
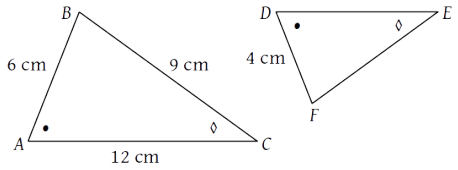


Similar Triangles

Recap

Determine $|EF|$.



Similar Triangles

AB and DF are corresponding sides, as are BC and EF.

$$\frac{|EF|}{|BC|} = \frac{|DF|}{|AB|}$$
$$\frac{|EF|}{9} = \frac{4}{6}$$
$$6|EF| = 36$$
$$|EF| = 6 \text{ cm}$$

Applications of Similar Triangles

Many situations can be modelled using similar triangles.

For example, the heights of inaccessible objects (cliffs, trees, etc.) can be estimated using smaller models.

As long as we are able to establish a proportion with three known quantities, we can solve for a fourth quantity.

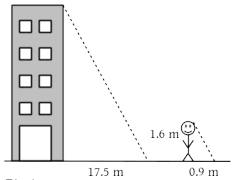
Of course, this only applies if the triangles are similar. In some cases, we need to make some assumptions that may not be 100% accurate in order to ensure this.

Applications of Similar Triangles

Example

At 3:00 pm, a building casts a shadow 17.5 m long. At the same time, a 1.6 m student casts a shadow 0.9 m long. Approximately how tall is the building?

Assuming the angle made by the sun is the same for both the building and the student (it isn't, but it's close enough), we can draw two triangles that are similar due to AA$\sim$.



Applications of Similar Triangles

Set up a proportion to solve, where b is the height of the building.

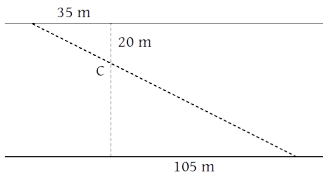
$$\frac{b}{17.5} = \frac{1.6}{0.9}$$
$$0.9b = 17.5 \times 1.6$$
$$0.9b = 28$$
$$b \approx 31.1$$

Therefore, the building is approximately 31.1 m tall.

Applications of Similar Triangles

Example

A channel marker in a river is located 20 m out from one shore. From a point 35 m down shore, the line of sight to another point 105 m the other way on the opposite shore passes through the marker, as shown. How wide is the river?



Applications of Similar Triangles

Assuming the shores are parallel (probably not completely true), the triangles are similar due to AA$\sim$. Set up a proportion to determine the distance, d, from the marker to the opposite shore.

$$\frac{d}{20} = \frac{105}{35}$$
$$d = 20 \times 3$$
$$d = 60$$

Since the channel marker is 20 m from one shore, and 60 m from the other, the river is $20 + 60 = 80$ m wide.

Applications of Similar Triangles

Example

Two corresponding sides in two similar triangles have lengths of 10 cm and 25 cm. If the perimeter of the smaller triangle is 48 cm, what is the perimeter of the larger one?

Since $\frac{25}{10} = 2.5$, the dimensions of the larger triangle are 2.5 times larger than those of the smaller one.

Since all sides are enlarged proportionally, the perimeter will also be 2.5 times larger.

Therefore, the perimeter of the larger triangle is $48 \times 2.5 = 120$ cm.

Applications of Similar Triangles

Example

A triangular garden with side lengths of 4 m, 5 m and 6 m is enlarged so that it holds ten times as many flowers. What are its new dimensions?

We can find the new dimensions of the garden if we can determine the scale factor, k.

Recall that the area increases by a factor of k^2.

Therefore, if the area increases ten times, then $k^2 = 10$, and the scale factor is $k = \sqrt{10}$.

Applications of Similar Triangles

The new dimensions are equal to the original values multiplied by $\sqrt{10}$.

$$4 \times \sqrt{10} \approx 12.65 \quad 5 \times \sqrt{10} \approx 15.81 \quad 6 \times \sqrt{10} \approx 18.97$$

The new dimensions are approximately 12.65 m, 15.81 m and 18.97 m.

Questions?