MPM2D: Principles of Mathematics

Applications of Similar Triangles

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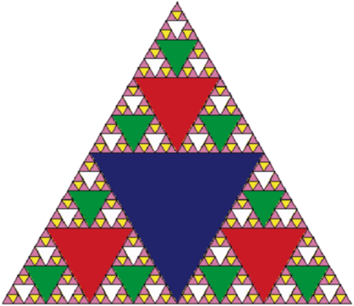
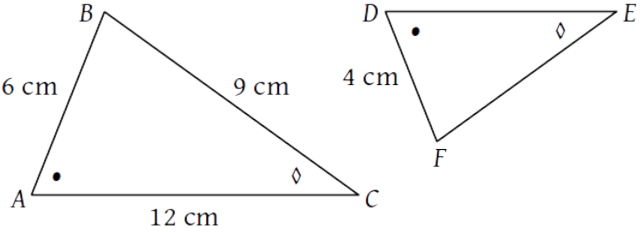


Similar Triangles

Recap

Determine $|EF|$.



Similar Triangles

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$$\frac{|EF|}{|BC|} = \frac{|DF|}{|AB|}$$

$$\frac{|EF|}{9} = \frac{4}{6}$$

$$6|EF| = 36$$

$$|EF| = 6 \text{ cm}$$

Applications of Similar Triangles

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For example, the heights of inaccessible objects (cliffs, trees, etc.) can be estimated using smaller models.

As long as we are able to establish a proportion with three known quantities, we can solve for a fourth quantity.

Of course, this only applies if the triangles are similar. In some cases, we need to make some assumptions that may not be 100% accurate in order to ensure this.

Applications of Similar Triangles

Example

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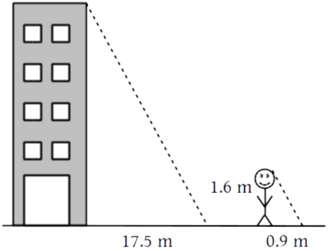
Assuming the angle made by the sun is the same for both the building and the student (it isn't, but it's close enough), we can draw two triangles that are similar due to AA$\sim$.



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$$\begin{aligned}\frac{b}{17.5} &= \frac{1.6}{0.9}\\ 0.9b &= 17.5 \times 1.6\\ 0.9b &= 28\\ b &\approx 31.1\end{aligned}$$

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Therefore, the building is approximately 31.1 m tall.

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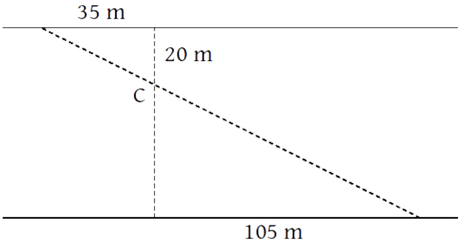
A channel marker in a river is located 20 m out from one shore. From a point 35 m down shore, the line of sight to another point 105 m the other way on the opposite shore passes through the marker, as shown. How wide is the river?



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Assuming the shores are parallel (probably not completely true), the triangles are similar due to AA$\sim$. Set up a proportion to determine the distance, d, from the marker to the opposite shore.

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$$\frac{d}{20} = \frac{105}{35}$$
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$$\begin{aligned} \frac{d}{20} &= \frac{105}{35} \\ d &= 20 \times 3 \\ d &= 60 \end{aligned}$$

Since the channel marker is 20 m from one shore, and 60 m from the other, the river is $20 + 60 = 80$ m wide.

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Therefore, the perimeter of the larger triangle is $48 \times 2.5 = 120$ cm.

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We can find the new dimensions of the garden if we can determine the scale factor, k.

Recall that the area increases by a factor of k^2.

Therefore, if the area increases ten times, then $k^2 = 10$, and the scale factor is $k = \sqrt{10}$.

Applications of Similar Triangles

The new dimensions are equal to the original values multiplied by $\sqrt{10}$.

$$4 \times \sqrt{10} \approx 12.65 \quad 5 \times \sqrt{10} \approx 15.81 \quad 6 \times \sqrt{10} \approx 18.97$$

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The new dimensions are approximately 12.65 m, 15.81 m and 18.97 m.

Questions?