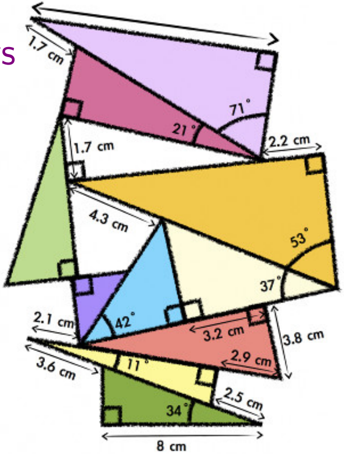
MPM2D: Principles of Mathematics

Applications of Sine/Cosine Laws

J. Garvin



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For right triangles, do not forget about simpler tools: Pythagorean Theorem, primary trigonometric ratios, and inverse ratios.

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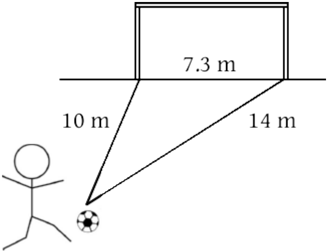
Sketch a diagram as shown.



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So, the player must shoot the ball through a 30° angle to score a goal.

Applications of Sine and Cosine Laws

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A pilot wishes to fly an airplane due East, but a strong wind blowing Southeast at 100 km/h keeps blowing the airplane off-course. If the airplane has a cruising speed of 450 km/h, in what direction should the pilot fly to reach the destination?

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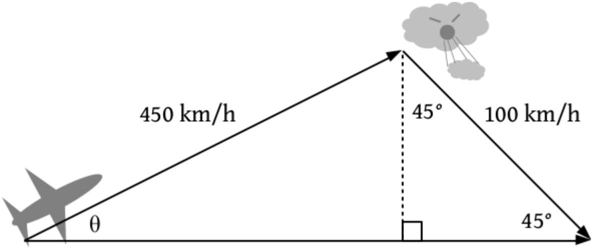
A diagram, showing the desired angle θ, is below.



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The pilot should fly approximately 9° North of East.

Applications of Sine and Cosine Laws

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Two surveyors, Alice and Bob, need to determine the height of a steep cliff. They stand 50 m apart where they each have a clear view of the cliff and each other. Bob measures an angle of elevation of 61° from the base of the cliff to its highest point. He also measures the angle between Alice and the base of the cliff as 72°. Alice measures the angle between Bob and the base of the cliff as 38°. How tall is the cliff?

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In complex situations like this, it is always important to draw an accurate diagram labelled with all given information.

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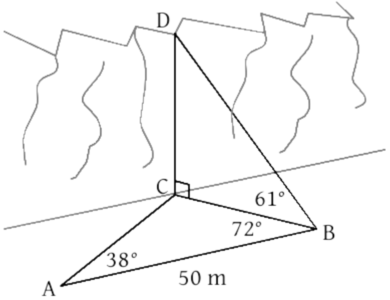
In the diagram below, ΔABC lies horizontal on the ground, while ΔBCD projects vertically.



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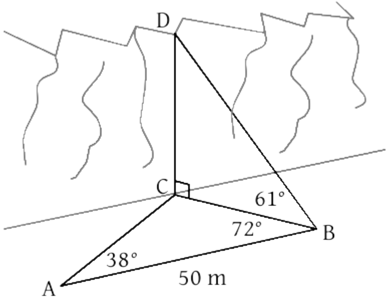

The height of the cliff is $|CD|$, but there is not enough information in the vertical triangle to solve yet.

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So, the height of the cliff is approximately 59.1 m.

Questions?