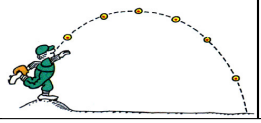
MPM2D: Principles of Mathematics

Applications of Quadratic Relations

Part 1: Roots Word Problems

J. Garvin



Solving Quadratic Equations

Recap

Solve $3x^2 - 8x + 1 = 0$ using the quadratic formula.

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(1)}}{2(3)}$$
$$x = \frac{8 \pm \sqrt{52}}{6}$$
$$x = \frac{8 \pm 2\sqrt{13}}{6}$$
$$x = \frac{4 \pm \sqrt{13}}{3}$$

Applications of Quadratic Relations

Many applications can be modelled by quadratic relations, including:

- projectile motion,
- geometry and measurement,
- optics,
- electronics,
- revenue and profit,
- and many, many other things.

In many cases, we need to solve a quadratic equation for a particular value. That is, determine the value of the independent variable.

We can do this by using any of the three methods covered in class: factoring, completing the square, and using the quadratic formula.

Applications of Quadratic Relations

Example

A ball is launched into the air with a velocity of 30 m/s, from an initial height of 2 m. Its height, h metres, is given by the equation $h = -4.9t^2 + 30t + 2$, where t is the time in seconds. How long does it take for it to hit the ground?

Upon impact, $h = 0$. Use the quadratic formula to solve.

$$t = \frac{-30 \pm \sqrt{30^2 - 4(-4.9)(2)}}{2(-4.9)}$$
$$t = \frac{30 \pm \sqrt{939.2}}{9.8}$$
$$t \approx -0.066 \text{ or } 6.188$$

Since we do not have a time machine, we reject the negative time. The ball hits the ground after approximately 6.2 sec.

Applications of Quadratic Relations

Example

Determine two integers that have a sum of 413 and a product of 28 362.

Let x and y be the two integers. Then $x + y = 413$ and $xy = 28\,362$.

Rearranging the first equation gives $y = 413 - x$, which we can substitute into the second equation.

$$xy = 28\,362$$
$$x(413 - x) = 28\,362$$
$$x^2 - 413x + 28\,362 = 0$$

While this is solvable by factoring, it is probably easier to complete the square or use the quadratic formula.

Applications of Quadratic Relations

$$x = \frac{-(-413) \pm \sqrt{(-413)^2 - 4(1)(28\,362)}}{2(1)}$$
$$x = \frac{413 \pm \sqrt{57\,121}}{2}$$
$$x = \frac{413 \pm 239}{2}$$
$$x = 326 \text{ or } 87$$

When $x = 326$, $y = 413 - 326 = 87$, and when $x = 87$, $y = 413 - 87 = 326$. The order does not matter.

Therefore, the two integers are 87 and 326.

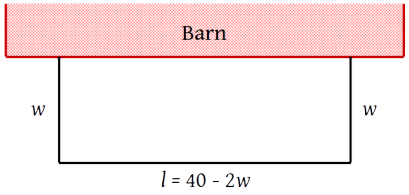
We can verify this by noting that $87 \times 326 = 28\,362$.

Applications of Quadratic Relations

Example

A farmer has 40 m of fencing to build three sides of a rectangular pen that will be built against one side of a 20 m long barn. What dimensions will produce an area of 150 m^2?

If w represents the width of the pen, then the farmer has $l = 40 - 2w$ metres of fencing to build the remaining side.



Applications of Quadratic Relations

Since $A = lw$, an expression for the area of the pen is $A = w(40 - 2w)$, or $A = -2w^2 + 40w$.

Substitute $A = 150$ and solve.

$$\begin{aligned} 150 &= -2w^2 + 40w \\ 0 &= -2w^2 + 40w - 150 \\ 0 &= w^2 - 20w + 75 \\ 0 &= (w - 5)(w - 15) \\ w &= 5 \text{ or } 15 \end{aligned}$$

If the width is 5 m, then the length is $l = 40 - 2(5) = 30$ m. This is too long for the barn, so we reject this solution.

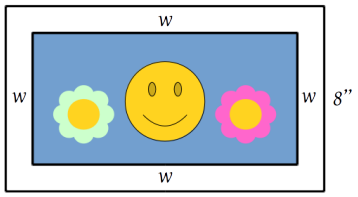
If the width is 15 m, the length is $l = 40 - 2(15) = 10$ m. Thus, the dimensions are 15×10 m.

Applications of Quadratic Relations

Example

An 8×10 in photograph is reduced in size so that it is centred on the page, with a border of uniform width around it. If the area of the reduced photograph is 63 in^2, determine the width of the border.

If w is the width of the border, we obtain the diagram below.



Applications of Quadratic Relations

Substitute $A = 63$ into the area formula.

$$\begin{aligned} 63 &= (8 - 2w)(10 - 2w) \\ 63 &= 4w^2 - 36w + 80 \\ 0 &= 4w^2 - 36w + 17 \\ 0 &= 4w^2 - 2w - 34w + 17 \\ 0 &= (2w - 1)(2w - 17) \\ w &= \tfrac{1}{2} \text{ or } \tfrac{17}{2} \end{aligned}$$

We reject $\frac{17}{2}$, since a border with this width is too wide for the photograph.

Therefore, the width of the border is $\frac{1}{2}$ of an inch.

Questions?