MPM2D: Principles of Mathematics

Right-Angled Trigonometry

Applications Using Multiple Triangles

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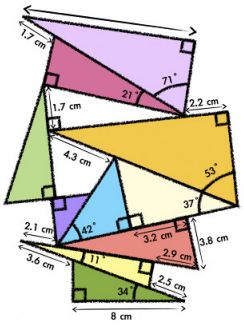
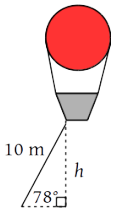


Applications of Trigonometric Ratios

Recap

A 10 m rope tethering a hot air balloon to the ground has an angle of elevation of 78°. How high above the ground is the balloon?



In the diagram, h is the height of the balloon.

$$\sin 78^\circ = \frac{h}{10}$$
$$h = 10 \times \sin 78^\circ$$
$$h \approx 9.78 \text{ m}$$

Applications of Trigonometric Ratios

Sometimes a situation involves two or more right triangles, each of which share a common side or angle with another.

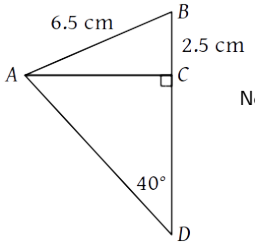
In these cases, we can use information from one triangle to find the measure of the common side or angle, then use that measure to find additional information in another.

Any combination of trigonometric ratios, inverse trigonometric ratios, or Pythagorean Theorem may be used.

Applications of Trigonometric Ratios

Example

Determine $|AD|$.



First, find $|AC|$ using the Pythagorean Theorem.

$$|AC|^2 + 2.5^2 = 6.5^2$$
$$|AC| = 6 \text{ cm}$$

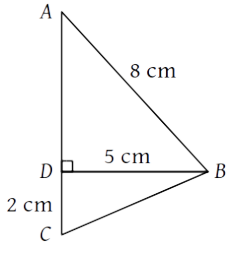
Now, find $|AD|$ using sine.

$$\sin 40^\circ = \frac{6}{|AD|}$$
$$|AD| = \frac{6}{\sin 40^\circ}$$
$$|AD| \approx 9.3 \text{ cm}$$

Applications of Trigonometric Ratios

Example

Determine the measure of $\angle ABC$.



Use cosine to find $\angle ABD$.

$$\cos ABD = \frac{5}{8}$$
$$ABD = \cos^{-1}\left(\frac{5}{8}\right)$$
$$ABD = 51.3^\circ$$

Use tangent to find $\angle DBC$.

$$\tan DBC = \frac{2}{5}$$
$$DBC = \tan^{-1}\left(\frac{2}{5}\right)$$
$$DBC \approx 21.8^\circ$$

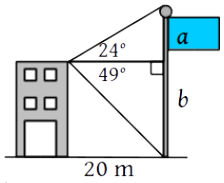
$$\angle ABC \approx 51.3 + 21.8 \approx 73.1^\circ.$$

Applications of Trigonometric Ratios

Example

A very tall flagpole stands 20 m in front of a school. From the roof of the school, the angle of elevation to the top of the flagpole is 24°, while the angle of depression to the base of the flagpole is 49°. How tall is the flagpole?

A diagram looks something like below, where a is the vertical distance above the roof, and b the vertical distance below.



Applications of Trigonometric Ratios

Use the tangent ratio twice, to find a and b.

$$\tan 24^\circ = \frac{a}{20} \qquad \tan 49^\circ = \frac{b}{20}$$
$$a = 20 \times \tan 24^\circ \qquad b = 20 \times \tan 49^\circ$$
$$a \approx 8.9 \text{ m} \qquad b \approx 23.0 \text{ m}$$

The height of the flagpole is the sum of the vertical distances, or $8.9 + 23.0 \approx 31.9$ m.

Questions?