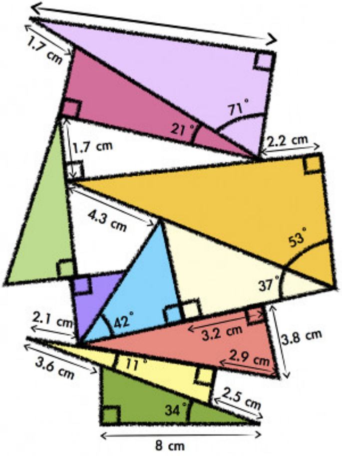
MPM2D: Principles of Mathematics

Right-Angled Trigonometry
Applications Using Multiple Triangles

J. Garvin



Applications of Trigonometric Ratios

Recap

A 10 m rope tethering a hot air balloon to the ground has an angle of elevation of 78°. How high above the ground is the balloon?

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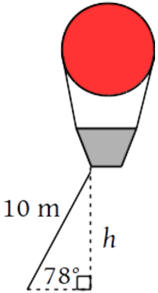

In the diagram, h is the height of the balloon.

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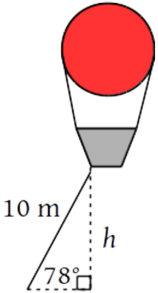

In the diagram, h is the height of the balloon.

$$\sin 78^\circ = \frac{h}{10}$$

$$h = 10 \times \sin 78^\circ$$

$$h \approx 9.78 \text{ m}$$

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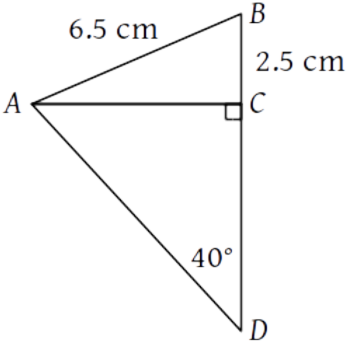
Any combination of trigonometric ratios, inverse trigonometric ratios, or Pythagorean Theorem may be used.

Applications of Trigonometric Ratios

Example

Determine $|AD|$.



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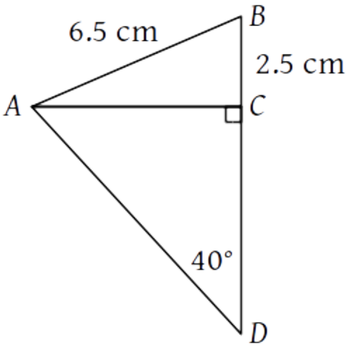

First, find $|AC|$ using the Pythagorean Theorem.

$$|AC|^2 + 2.5^2 = 6.5^2$$
$$|AC| = 6 \text{ cm}$$

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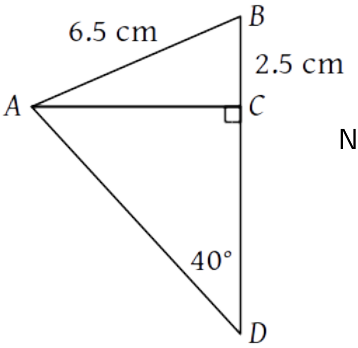


First, find $|AC|$ using the Pythagorean Theorem.

$$|AC|^2 + 2.5^2 = 6.5^2$$
$$|AC| = 6 \text{ cm}$$

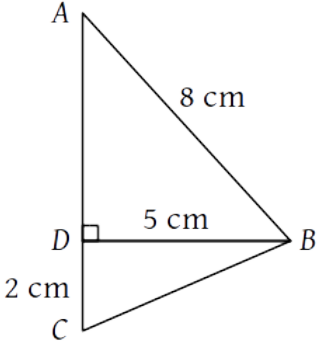
Now, find $|AD|$ using sine.

$$\sin 40^\circ = \frac{6}{|AD|}$$
$$|AD| = \frac{6}{\sin 40^\circ}$$
$$|AD| \approx 9.3 \text{ cm}$$

Applications of Trigonometric Ratios

Example

Determine the measure of $\angle ABC$.



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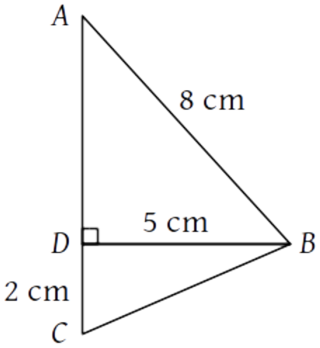

Use cosine to find $\angle ABD$.

$$\cos ABD = \frac{5}{8}$$
$$ABD = \cos^{-1}\left(\frac{5}{8}\right)$$
$$ABD = 51.3^\circ$$

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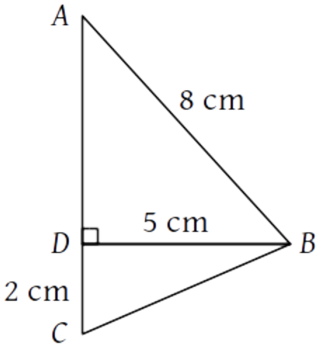

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$$\begin{aligned} \cos ABD &= \tfrac{5}{8} \\ ABD &= \cos^{-1}\left(\tfrac{5}{8}\right) \\ ABD &= 51.3^\circ \end{aligned}$$

Use tangent to find $\angle DBC$.

$$\begin{aligned} \tan DBC &= \tfrac{2}{5} \\ DBC &= \tan^{-1}\left(\tfrac{2}{5}\right) \\ DBC &\approx 21.8^\circ \end{aligned}$$

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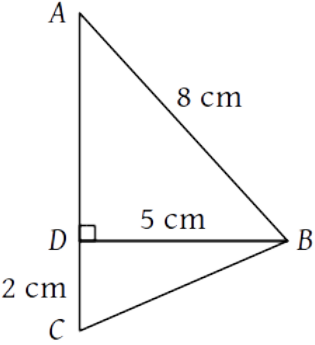

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$\angle ABC \approx 51.3 + 21.8 \approx 73.1^\circ$.

Applications of Trigonometric Ratios

Example

A very tall flagpole stands 20 m in front of a school. From the roof of the school, the angle of elevation to the top of the flagpole is 24°, while the angle of depression to the base of the flagpole is 49°. How tall is the flagpole?

Applications of Trigonometric Ratios

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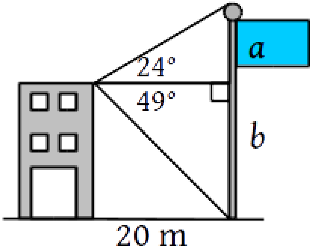
A very tall flagpole stands 20 m in front of a school. From the roof of the school, the angle of elevation to the top of the flagpole is 24°, while the angle of depression to the base of the flagpole is 49°. How tall is the flagpole?

A diagram looks something like below, where a is the vertical distance above the roof, and b the vertical distance below.



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Use the tangent ratio twice, to find a and b.

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$$\begin{aligned} \tan 24^\circ &= \frac{a}{20} \\ a &= 20 \times \tan 24^\circ \\ a &\approx 8.9 \text{ m} \end{aligned}$$

$$\begin{aligned} \tan 49^\circ &= \frac{b}{20} \\ b &= 20 \times \tan 49^\circ \\ b &\approx 23.0 \text{ m} \end{aligned}$$

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The height of the flagpole is the sum of the vertical distances, or $8.9 + 23.0 \approx 31.9$ m.

Questions?