Rates
Recap
A store sells orange juice in two sizes: 1.8 L for $2.50 or 3.5 L for $4.25. Which represents the better bargain?

The unit rate for the 1.8 L bottle is $2.50 \div 1.8 \approx 1.39$ $\$/L, while it is $4.25 \div 3.5 \approx 1.21$ $\$/L for the 3.5 L bottle.

Assuming no juice is wasted, the better bargain is the 3.5 L bottle.

Exponents

Recall that an exponent indicates repeated multiplication of a value.

For instance, $5^2$ is the same as $5 \times 5$, while $3^4$ is the same as $3 \times 3 \times 3 \times 3$.

Scientific calculators have buttons for exponentiation, typically labelled something like $x^y$, $y^x$, or simply $\hat{}$. There may also be shortcuts for common exponents, such as $x^2$ or $x^3$.

Since values are being multiplied, exponentiation can result in very large (or small) values.

Example

Express $7 \times 7 \times 7$ using an exponent.

Since 7 is multiplied three times, $7 \times 7 \times 7$ can be written with an exponent as $7^3$.

Example

Simplify, then evaluate, $2 \times 2 \times 2 \times 2 \times 2$.

$2 \times 2 \times 2 \times 2 \times 2 = 2^5$, or 32.

Example

Simplify, then evaluate, $1.8 \times 1.8 \times 1.8 \times 1.8$.

Exponentiation can be done with decimal values in the same way as it is done with integers.

$1.8 \times 1.8 \times 1.8 \times 1.8 = 1.8^4$, or 10.4976.

Fractions and Exponents

What about $\left(\frac{2}{3}\right)^2$?

Recall that $\left(\frac{2}{3}\right)^2$ is the same as $\frac{2}{3} \times \frac{2}{3}$.

Multiplying, we get $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$.

Since $2^2 = 4$ and $3^2 = 9$, the result was that both the numerator and denominator were squared.

In general, we can apply an exponent to each component (numerator or denominator) individually.
**Exponents**

Example
Evaluate \((\frac{2}{5})^3\).

Since \(2^3 = 8\) and \(5^3 = 125\), \((\frac{2}{5})^3 = \frac{8}{125}\).

Example
Evaluate \((\frac{1}{10})^6\).

Since \(1^6 = 1\) and \(10^6 = 10,000,000\), \((\frac{1}{10})^6 = \frac{1}{10,000,000}\).

**Negative Exponents**

Negative values can also be raised to an exponent.

Example
Evaluate \((-5)^3\).

\((-5)^3 = \(-5\) \times \(-5\) \times \(-5\) = \(-125\).

Example
Evaluate \(-2.5^4\).

Since the exponent does not apply to the negative, \(-2.5^4 = -39.0625\).

**Negative Exponents**

We can make some generalizations about the sign of an exponentiated value by examining both the value and the exponent.

Multiplying two negatives produces a positive, multiplying three negatives produces a negative, multiplying four negatives produces a positive, etc.

In general, if a negative value has an even exponent, then its final value will be positive.

If a negative value has an odd exponent, then its final value will be negative.

**Working with Exponents**

Example
Evaluate \(2^3 \times 7^2\).

According to the order of operations, exponentiation precedes multiplication.

\[2^3 \times 7^2 = 8 \times 49 = 392\]
Working with Exponents

Example
Evaluate $3^2 \times 3^3$.

As before, exponentiate first.

$$3^2 \times 3^3 = 9 \times 27$$
$$= 243$$

Note that $243 = 3^5$, and that $3^{2+3} = 3^5$. We will cover this result in more detail in the next lesson.

Questions?