MPM1D: Principles of Mathematics

Solving Equations

Part 1: Simple Equations

J. Garvin

Solving Equations

To *solve* an equation means to find any value(s) that cause that equation to be true.

This may involve one variable, in the case of $x + 7 = 9$, or more than one variable, such as $3x - 2y = 5$.

Formally, we might say that such value(s) *satisfy* the equation.

There are three main methods that can be used to solve an equation.

1. by inspection,
2. by balancing the equation using a graphical method, or
3. by using algebra to apply opposite operations

Solving By Inspection

Example

Solve $x + 1 = 4$ by inspection.

Using some basic number sense, it is probably fairly easy to see that if $x = 3$, then $3 + 1 = 4$.

Since $x = 3$ satisfies the equation, the solution is $x = 3$.

Note that in later courses, it may be possible for more than one value to satisfy an equation. In this case, inspection may not be the best approach, since it may result in you missing possible solutions.

Solving By Balancing Graphically

A graphical way of solving an equation relies on the idea of keeping things *in balance*.

Consider the equation $x + 5 = 8$. The equals sign indicates that the value on the left, $x + 5$, is equivalent to the value on the right, 8.

If we were to imagine these values on a scale, it would be balanced.

Any change to one side of the equation *must* also be applied to the other side, or the equation will no longer be balanced.

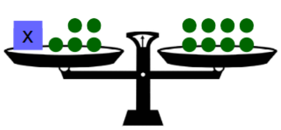
For example, if we were to add 1 to the left hand side to produce $x + 6$, we would also need to add 1 to the right hand side, producing 9.

Solving By Balancing Graphically

Example

Solve $x + 5 = 8$ graphically.

Begin by placing one x and 5 positive marks on the left hand side and 8 positive marks on the right hand side, as shown.



Solving By Balancing Graphically

A positive mark on the left can be removed if it has a matching positive mark on the right. Thus, we can remove 5 positive marks from the left and from the right to keep things in balance.

As x is by itself, the 3 marks on the right must be equivalent to the x on the left.

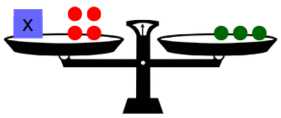
Therefore, the solution is $x = 3$.

Solving By Balancing Graphically

Example

Solve $x - 4 = 3$ graphically.

Begin by placing one x and 4 negative marks (red) on the left hand side and 3 positive marks (green) on the right hand side.

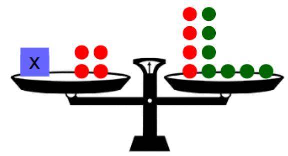


Solving By Balancing Graphically

Since there are no negative marks on the right hand side, we need to add them before we can cancel the ones on the left.

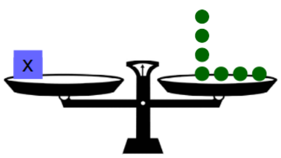
To do this, use the concept of "zero pairs" – a positive value and its negative equivalent (e.g. 3 and -3) will cancel when added together.

Therefore, we can add both a positive and a negative mark to one side of the equation without changing its value, since we are effectively adding zero.



Solving By Balancing Graphically

Now we can cancel the four negative marks on each side, resulting in the figure below.



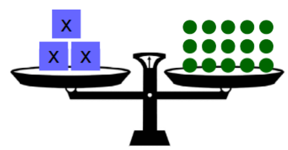
Since x is now by itself, the answer is $x = 7$.

Solving By Balancing Graphically

Example

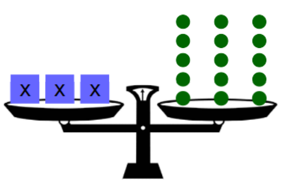
Solve $3x = 15$ graphically.

Place 3 xs on the left hand side and 15 positive marks on the right hand side, as shown.



Solving By Balancing Graphically

Split the right hand side into 3 equal groups to match the xs.



Each group contains 5 positive marks, so the solution is $x = 5$.

Solving Algebraically (Opposite Operations)

The previous methods have limitations.

Inspection is really only good for very simple equations, when it is "obvious" what value works.

Balancing an equation graphically takes time, especially when values are large, and can be confusing if values do not split evenly into groups.

A "better" approach is to use algebra to solve an equation. Specifically, we can apply *opposite operations* until x is by itself, or *isolated*.

Solving Algebraically (Opposite Operations)

Example

Solve $x - 10 = 14$ algebraically.

Using the previous concept of keeping things in balance, we can add 10 to both the left hand side and the right hand side of the equation.

$$x - 10 + 10 = 14 + 10$$

Both -10 and 10 cancel on the left hand side.

$$\begin{aligned} x - \cancel{10} + \cancel{10} &= 14 + 10 \\ x &= 24 \end{aligned}$$

Since x is isolated, the solution is $x = 24$.

Solving Algebraically (Opposite Operations)

Example

Solve $x + 19 = -13$ algebraically.

Subtract 19 from both sides of the equation to cancel out the 19 on the left.

$$\begin{aligned} x + \cancel{19} - \cancel{19} &= -13 - 19 \\ x &= -32 \end{aligned}$$

The solution is $x = -32$. We can check our answer by *substituting* back into the original equation.

$$\begin{aligned} x + 19 &= -32 + 19 \\ &= -13 \end{aligned}$$

Since we obtain the value on the right hand side, our answer is correct.

Solving Algebraically (Opposite Operations)

Example

Solve $3x = 51$ algebraically.

The "opposite" operation to multiplication is division, so we can divide both sides of the equation by 3 to isolate x.

$$\frac{3x}{3} = \frac{51}{3}$$

Recall that any value divided by itself is 1, so the 3s cancel.

$$\begin{aligned} \frac{\cancel{3}x}{\cancel{3}} &= \frac{51}{3} \\ x &= 17 \end{aligned}$$

The solution is $x = 17$, since $3 \times 17 = 51$.

Solving Algebraically (Opposite Operations)

Example

Solve $\frac{x}{4} = 13$ algebraically.

Remember that $\frac{x}{4}$ indicates division, so we need to multiply to isolate x.

$$\begin{aligned} \frac{x}{4} &= 13 \\ \frac{\cancel{4}x}{\cancel{4}} &= 13 \times 4 \\ x &= 52 \end{aligned}$$

The solution is $x = 52$, since $\frac{52}{4} = 13$.

Questions?