MPM1D: Principles of Mathematics

Solving Equations

Part 3: Equations Involving Fractions

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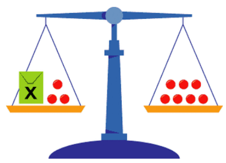

Solving Multi-Step Equations

Recap

Solve $5(x-4)=2(x+11)$ algebraically.

Use the distributive property to expand, collect like terms, then solve.

$$\begin{aligned}5x-20&=2x+22\\5x-2x&=22+20\\3x&=42\\\frac{\cancel{3}x}{\cancel{3}}&=\frac{42}{3}\\x&=14\end{aligned}$$

Solving Equations Involving Fractions

Sometimes equations involve fractions, which may be written in different ways.

For example, an equation may be either $\frac{1}{5}(x+6)=8$, or $\frac{x+6}{5}=8$.

Both of these forms represent the same equation, and are merely written differently (likely as a matter of preference).

Remember that a fraction simply represents a division by some quantity, so to apply an "opposite operation" to a division generally involves a multiplication.

Solving Equations Involving Fractions

Example

Solve $\frac{x}{4}=7$.

The "opposite" operation to division is multiplication, so multiply both sides of the equation by 4.

$$\begin{aligned}\frac{\cancel{4}\cdot x}{\cancel{4}}&=4\cdot 7\\x&=28\end{aligned}$$

Since $28\div 4=7$, $x=28$ is the solution.

Solving Equations Involving Fractions

Example

Solve $\frac{1}{2}(x-5)=13$.

Here, the binomial on the left hand side is being multiplied by $\frac{1}{2}$, which is equivalent to it being divided by 2, so multiply both sides by 2.

$$\begin{aligned}\cancel{2}\cdot\frac{\cancel{1}}{\cancel{2}}(x-5)&=2\cdot 13\\x-5&=26\\x&=26+5\\x&=31\end{aligned}$$

Note that this equation may have also been expressed as $\frac{x-5}{2}=13$.

Solving Equations Involving Fractions

Example

Solve $\frac{3}{4}(5x-1)=-12$.

As before, multiply by the denominator, 4, but this time the numerator will remain.

$$\begin{aligned}\cancel{4}\cdot\frac{3}{\cancel{4}}(5x-1)&=-12\cdot 4\\3(5x-1)&=-48\\15x-3&=-48\\15x&=-48+3\\15x&=-45\\\frac{\cancel{15}x}{\cancel{15}}&=-\frac{45}{15}\\x&=-3\end{aligned}$$

Solving Equations Involving Fractions

Example

Solve $\frac{1}{3}(x+22) = \frac{1}{4}(x+42)$.

Since the LCM of 3 and 4 is 12, multiplying both sides by 12 will eliminate the fractions.

$$\begin{aligned} 12 \cdot \tfrac{1}{3}(x+22) &= 12 \cdot \tfrac{1}{4}(x+42) \\ 4(x+22) &= 3(x+42) \\ 4x + 88 &= 3x + 126 \\ 4x - 3x &= 126 - 88 \\ x &= 38 \end{aligned}$$

This question could also have been solved by expanding first, but would have required more work with fractions.

Solving Equations Involving Fractions

Example

Solve $\dfrac{x+7}{6} = \dfrac{3x-2}{9}$.

The LCM of 6 and 9 is 18, so multiplying both sides by 18 will eliminate the fractions.

$$\begin{aligned} 18 \cdot \frac{x+7}{6} &= 18 \cdot \frac{3x-2}{9} \\ 3(x+7) &= 2(3x-2) \\ 3x + 21 &= 6x - 4 \\ 3x - 6x &= -4 - 21 \\ -3x &= -25 \\ \frac{-\cancel{3}x}{-\cancel{3}} &= \frac{-25}{-3} \\ x &= \tfrac{25}{3} \end{aligned}$$

Questions?