Solving Multi-Step Equations

Recap
Solve \(5(x - 4) = 2(x + 11)\) algebraically.

Use the distributive property to expand, collect like terms, then solve.
\[
5x - 20 = 2x + 22
\]
\[
5x - 2x = 22 + 20
\]
\[
3x = 42
\]
\[
\frac{3x}{3} = \frac{42}{3}
\]
\[
x = 14
\]

Solving Equations Involving Fractions

Sometimes equations involve fractions, which may be written in different ways.

For example, an equation may be either \(\frac{1}{2}(x + 6) = 8\), or \(x + 6 = 8\).

Both of these forms represent the same equation, and are merely written differently (likely as a matter of preference).

Remember that a fraction simply represents a division by some quantity, so to apply an “opposite operation” to a division generally involves a multiplication.

Example
Solve \(\frac{x}{4} = 7\).

The “opposite” operation to division is multiplication, so multiply both sides of the equation by 4.
\[
\frac{4 \cdot x}{4} = 4 \cdot 7
\]
\[
x = 28
\]

Since \(28 \div 4 = 7\), \(x = 28\) is the solution.

Example
Solve \(\frac{3}{4}(x - 1) = -12\).

As before, multiply by the denominator, 4, but this time the numerator will remain.
\[
4 \cdot \frac{3}{4}(5x - 1) = -12 \cdot 4
\]
\[
3(5x - 1) = -48
\]
\[
15x - 3 = -48
\]
\[
15x = -48 + 3
\]
\[
15x = -45
\]
\[
\frac{15x}{\frac{15}{1}} = \frac{-45}{\frac{15}{1}}
\]
\[
x = -3
\]
**Solving Equations Involving Fractions**

**Example**

Solve \( \frac{1}{3}(x + 22) = \frac{1}{4}(x + 42) \).

Since the LCM of 3 and 4 is 12, multiplying both sides by 12 will eliminate the fractions.

\[
12 \cdot \frac{1}{3}(x + 22) = 12 \cdot \frac{1}{4}(x + 42) \\
4(x + 22) = 3(x + 42) \\
4x + 88 = 3x + 126 \\
4x - 3x = 126 - 88 \\
x = 38
\]

This question could also have been solved by expanding first, but would have required more work with fractions.

The LCM of 6 and 9 is 18, so multiplying both sides by 18 will eliminate the fractions.

\[
18 \cdot \frac{x + 7}{6} = 18 \cdot \frac{3x - 2}{9} \\
3(x + 7) = 2(3x - 2) \\
3x + 21 = 6x - 4 \\
3x - 6x = -4 - 21 \\
-3x = -25 \\
-3x = -25 \\
\frac{-3x}{-3} = \frac{-25}{-3} \\
x = \frac{-25}{3}
\]

**Questions?**