Slope as a Rate of Change

Recap

Determine the slopes of $AB$ and $CD$ below.

\[ \text{Slope of } AB = \frac{5}{3} = 2 \]
\[ \text{Slope of } CD = -\frac{3}{3} = -1 \]

Slope as a Rate of Change

\[ \text{A rate of change is a change in one quantity, relative to a change in another.} \]

Some examples:

- Speed measures the change in distance over a change in time.
- Acceleration measures the change in speed over a change in time.
- The area of a rectangle increases as its length increases.
- Air temperature decreases as altitude increases.

Rates of change are expressed as ratios, $\frac{\text{first quantity}}{\text{second quantity}}$.

Example

A cyclist rides 24 kilometres in 2 hours. What is the rate of change?

The rate of change is the cyclist’s speed, $\frac{24}{2} = 12$ km/h.

Example

The volume of air in a balloon leaks at a constant rate. At 2:00, the volume is 250 cm$^3$. At 4:00, the volume is 210 cm$^3$. What is the rate of change?

The rate of change is the decrease in air, $-\frac{40}{2} = -20$ cm$^3$/h. Note that the rate of change is negative, indicating a decrease.
The graph of the previous relation has a slope of 2, corresponding to the rate of change.

In this case, the rate of change is constant — it always changes by the same amount.

Thus, given a graph of a linear relation, we can determine its rate of change by determining its slope.

Example
The graph below shows a student’s distance from home as she walks back from school. Calculate the slope of the graph, and interpret it as a rate of change.

The rate of change is the student’s speed, 1.5 km/h. The negative sign indicates that the distance is decreasing.

Example
The distance-time graph below shows a student’s distance from a wall as he walks toward and away from it. Describe the student’s motion using rates of change.

For this example, let movement away from the wall be positive (distance is increasing), and movement toward the wall be negative (distance is decreasing).

At the beginning (at zero seconds), the student is 2 metres from the wall.

Over the next two seconds, the student walks to a distance 6 metres away from the wall. His speed is \( \frac{6}{2} = 3 \text{ m/s} \).

For the next three seconds, the student remains 6 metres from the wall. His speed is 0 m/s.

Finally, the student walks 6 metres toward the wall until he comes in contact with it. This takes 2 seconds, for a speed of \( -\frac{6}{2} = -3 \text{ m/s} \).

This last rate of change may be expressed as a negative value, or simply as “3 m/s toward the wall.”
Questions?