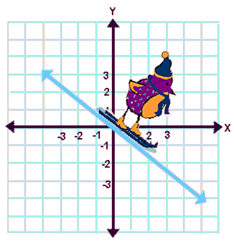
MPM1D: Principles of Mathematics

Slope as a Rate of Change

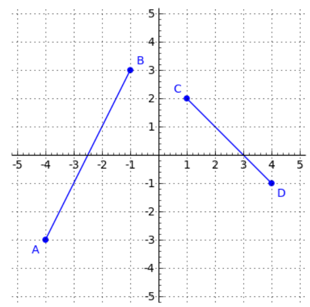
J. Garvin



Slope of a Line Segment

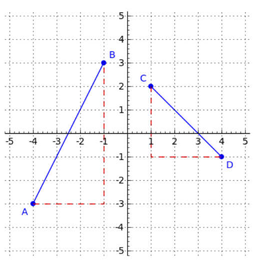
Recap

Determine the slopes of AB and CD below.



Slope of a Line Segment

Draw right triangles to determine the rise and the run.



Since AB moves upward, the slope is $\frac{6}{3} = 2$. Since CD moves downward, the slope is $-\frac{3}{3} = -1$.

Slope as a Rate of Change

A *rate of change* is a change in one quantity, relative to a change in another.

Some examples:

- speed measures the change in distance over a change in time.
- acceleration measures the change in speed over a change in time.
- the area of a rectangle increases as its length increases.
- air temperature decreases as altitude increases.

Rates of change are expressed as ratios, $\frac{\text{first quantity}}{\text{second quantity}}$.

Slope as a Rate of Change

Example

A cyclist rides 24 kilometres in 2 hours. What is the rate of change?

The rate of change is the cyclist's speed, $\frac{24}{2} = 12$ km/h.

Example

The volume of air in a balloon leaks at a constant rate. At 2:00, the volume is 250 cm^3. At 4:00, the volume is 210 cm^3. What is the rate of change?

The rate of change is the decrease in air, $-\frac{40}{2} = -20$ cm^3/h. Note that the rate of change is negative, indicating a decrease.

Slope as a Rate of Change

Consider a travelling fair that changes $5 admission and $2 per ride.

A table of values, showing the total cost, C, of attending the fair and riding n rides is below.

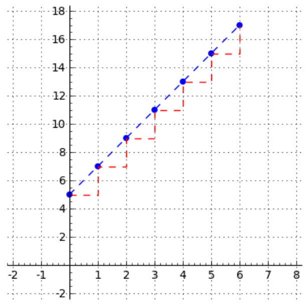
n	0	1	2	3	4	5	6
C	5	7	9	11	13	15	17

The cost increases by $2 for each additional ride.

How does this relate to a graph of the relation?

Slope as a Rate of Change

A graph of the relation is below.



Slope as a Rate of Change

The graph of the previous relation has a slope of 2, corresponding to the rate of change.

In this case, the rate of change is *constant* – it always changes by the same amount.

Slope as a Rate of Change

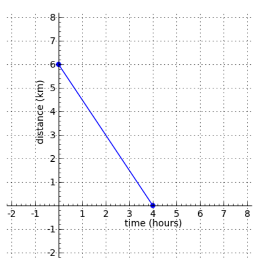
If a relation's rate of change is constant, it is a *linear relation*. The rate of change is represented by the slope of its graph.

Thus, given a graph of a linear relation, we can determine its rate of change by determining its slope.

Slope as a Rate of Change

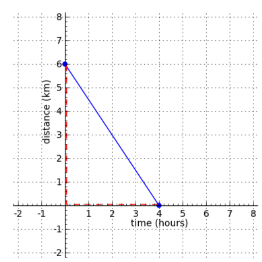
Example

The graph below shows a student's distance from home as she walks back from school. Calculate the slope of the graph, and interpret it as a rate of change.



Slope as a Rate of Change

The graph has a slope of $-\frac{6}{4} = -\frac{3}{2} = -1.5$.

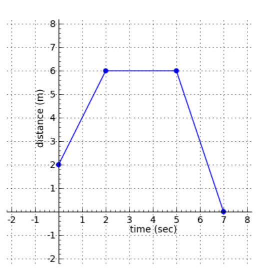


The rate of change is the student's speed, 1.5 km/h. The negative sign indicates that the distance is decreasing.

Slope as a Rate of Change

Example

The *distance-time* graph below shows a student's distance from a wall as he walks toward and away from it. Describe the student's motion using rates of change.



Slope as a Rate of Change

For this example, let movement away from the wall be positive (distance is increasing), and movement toward the wall be negative (distance is decreasing).

At the beginning (at zero seconds), the student is 2 metres from the wall.

Over the next two seconds, the student walks to a distance 6 metres away from the wall. His speed is $\frac{4}{2} = 2$ m/s.

For the next three seconds, the student remains 6 metres from the wall. His speed is 0 m/s.

Finally, the student walks 6 metres toward the wall until he comes in contact with it. This takes 2 seconds, for a speed of $-\frac{6}{2} = -3$ m/s.

This last rate of change may be expressed as a negative value, or simply as "3 m/s toward the wall."

Questions?