MPM1D: Principles of Mathematics

The Equation of a Line

Part 1: Slope-Intercept Form

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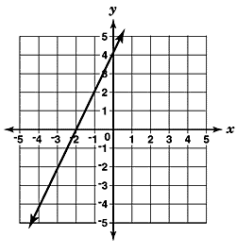


Slope-Intercept Form of a Line

In the last unit, we talked about relations that were either direct or partial variations.

Partial variations have the form $y = mx + b$, where m corresponds to the slope of the line, and b is the value of its y-intercept.

If $b = 0$, then the relation is a direct variation with equation $y = mx$. As before, m represents the slope of the line.

If we are given information about a line's slope and y-intercept, we can write its equation in the form $y = mx + b$.

Such an equation is said to be in *slope-intercept form*, since it contains those two pieces of information.

Slope-Intercept Form of a Line

Slope-Intercept Form of a Line

The equation of a line with a slope of m and a y-intercept of b has the equation $y = mx + b$, where (x, y) represent any point on the line.

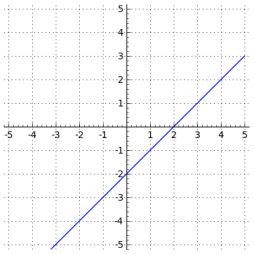
Given a graph, we can state the equation of the line by identifying the slope and the y-intercept.

This method is limited, however, if either the slope of y-intercept are difficult to make out.

Slope-Intercept Form of a Line

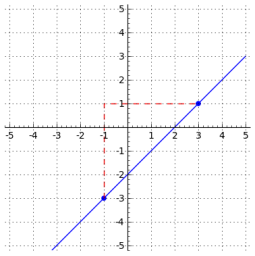
Example

What is the equation of the line below, expressed in slope-intercept form?



Slope-Intercept Form of a Line

From the graph, the slope of the line is $\frac{4}{4} = 1$ and the y-intercept is -2.

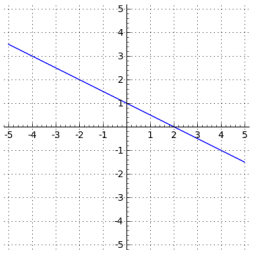


Therefore, an equation of the line is $y = 1x - 2$, or $y = x - 2$.

Slope-Intercept Form of a Line

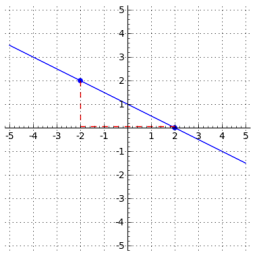
Example

What is the equation of the line below, expressed in slope-intercept form?



Slope-Intercept Form of a Line

From the graph, the slope of the line is $-\frac{2}{4} = -\frac{1}{2}$ and the y-intercept is 1.

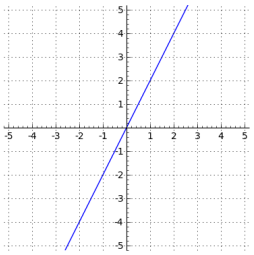


Therefore, an equation of the line is $y = -\frac{1}{2}x + 1$.

Slope-Intercept Form of a Line

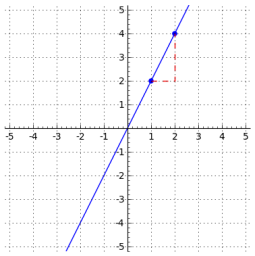
Example

What is the equation of the line below, expressed in slope-intercept form?



Slope-Intercept Form of a Line

From the graph, the slope of the line is $\frac{2}{1} = 2$ and the y-intercept is 0.



Therefore, an equation of the line is $y = 2x + 0$, or $y = 2x$.

Graphing Lines In Slope-Intercept Form

Graphing a line, given its equation in slope-intercept form, is generally a straightforward process.

Since we know the value of the y-intercept, "start" the line at that point.

From that point, count the rise and the run of the slope, ensuring that the line is rising in the case of a positive slope, or falling in the case of a negative slope.

Repeat as necessary, until enough points have been made on the graph and a straight line can be drawn through them.

Graphing Lines In Slope-Intercept Form

Example

Graph the line $y = -\frac{1}{4}x - 1$.

Since $b = -1$, we know that this is a partial variation with a y-intercept of -1, so we mark a point at $(0, -1)$.

From the y-intercept, we need to move to a new point using a slope of $\frac{1}{4}$.

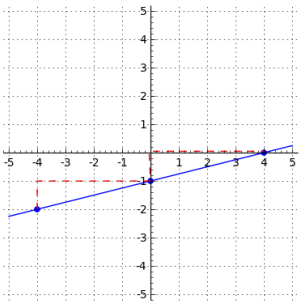
Thus, we rise up one unit, and move right four units. Note that this makes the line move upward, indicating a positive slope.

We can also move down one unit and left four units (reversing each direction) to maintain the direction of the line.

This puts us at $(4, 0)$ on the grid, and $(-4, -2)$ respectively.

Graphing Lines In Slope-Intercept Form

A graph of $y = \frac{1}{4}x - 1$ is below.



Graphing Lines In Slope-Intercept Form

Example

Graph the line $y = -\frac{3}{2}x$.

Since $b = 0$, we know that this is a direct variation, so the y-intercept is at the origin.

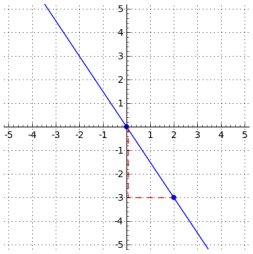
From the origin, we need to move to a new point using a slope of $-\frac{3}{2}$.

Thus, we move down three units, then move right two units. This makes the line move downward, indicating a negative slope.

This puts us at $(2, -3)$ on the grid.

Graphing Lines In Slope-Intercept Form

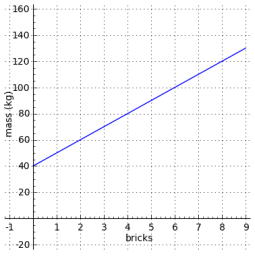
A graph of $y = -\frac{3}{2}x$ is below.



Interpreting the Graph of a Line

Example

The graph below shows the mass of a pallet of bricks. Interpret the graph, determine an equation for the relation, and calculate the mass of a pallet containing 200 bricks.



Interpreting the Graph of a Line

The graph has a vertical intercept of 40. Since the point $(0, 40)$ corresponds to a pallet containing no bricks, the mass of the pallet itself is 40 kg.

The slope of the graph shows an increase of 20 kg for every 2 bricks, for a slope of $\frac{20}{2} = 10$. This corresponds to a mass of 10 kg per brick.

An equation for the relation, then, is $M = 10b + 40$, where b is the number of bricks and M is the total mass of the bricks and the pallet.

A pallet containing 200 bricks would have a mass of $M = 10(200) + 40 = 2\,040$ kg.

Questions?