Order of Operations

Recap

Evaluate $(5 - 2) \times 4 \cdot 5^2$.

Remember to perform the subtraction and exponentiation before any multiplications.

$$(5 - 2) \times 4 \cdot 5^2 = 3 \times 4 \cdot 25 = 12 \cdot 25 = 300$$

Greatest Common Factor

Consider the numbers 4 and 10.
Both numbers are even, meaning they are both divisible by 2.
In fact, 2 is the largest value that divides evenly into both 4 and 10.
The largest value that divides evenly into two other values is known as the greatest common factor (GCF) of those values.

It may be useful to list all factors of each value to determine the GCF. Don’t forget 1 and the value itself as factors.

Reducing Fractions

While it is possible to express a fraction as $\frac{12}{20}$, as in “twelve out of twenty people...”, the same ratio can be expressed as $\frac{6}{10}$, or “six out of ten people...”.
The latter ratio is said to be reduced, since the values are smaller.

It is possible to reduce this ratio even further to $\frac{3}{5}$. This ratio is said to be in simplest form or lowest terms, since the same ratio cannot be expressed any smaller using integers.

Expressing fractions in lowest terms is a mathematical convention, and should be done whenever possible.

To reduce a fraction to lowest terms, both the numerator and the denominator should be divided by their greatest common factor.

Reducing Fractions

Example

Simplify \( \frac{21}{40} \).

Factors of 21 are 1, 3, 7, and 21, while factors of 40 are 1, 2, 4, 5, 8, 10, 20 and 40.

Since the GCF of 21 and 40 is 1, it is not possible to reduce the fraction any further.

In this case, the fraction is already in its simplest form, and no further work is necessary.

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Slide 7/19

Improper Fractions

When dealing with mixed fractions, such as \( 3\frac{1}{2} \), it is usually easier to convert them to improper fractions before multiplying or dividing.

Remember that the numerator of the improper fraction can be made by multiplying the denominator by the whole component, then adding the numerator of the mixed fraction.

The denominator of the improper fraction is the same as the denominator of the mixed fraction.

Thus, \( 3\frac{1}{2} \) becomes \( \frac{7}{2} \), since \( 2 \times 3 + 1 = 7 \).

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Slide 8/19

Multiplying Fractions

Example

Evaluate \( \frac{45}{22} \times \frac{77}{60} \).

Since the GCF of 2 and 20 is 2, and the GCF of 15 and 9 is 3, reduce these fractions first:

\[
\frac{\frac{2}{15} \times \frac{9}{20}}{\frac{1}{5} \times \frac{3}{10}} = \frac{3}{50}
\]

Compare this to multiplying first, then reducing:

\[
\frac{\frac{2}{15} \times \frac{9}{20}}{\frac{18}{300}} = \frac{3}{50}
\]

While both methods result in the same answer.

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Slide 12/19
Multiplying Fractions

If it is not easy to identify the GCF from two given values, then reducing by any factor will eventually produce the same result after multiple reductions.

For example, if we did not identify 15 as the GCF of 45 and 60, we might start by reducing each value by 5 instead.

\[
\frac{45}{22} \times \frac{77}{60} = \frac{9}{2} \times \frac{7}{12}
\]

Now reduce both 9 and 12 by 3.

\[
\frac{9}{2} \times \frac{7}{12} = \frac{3}{2} \times \frac{7}{4}
\]

\[
= \frac{21}{8}
\]

The answer is the same as earlier.

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Slide 13/19

Example

Evaluate \(\frac{2}{9} \times 6\).

Rewriting 6 as \(\frac{6}{1}\) may make it easier to multiply here. Don’t forget to reduce first.

\[
\frac{2}{9} \div \frac{3}{7} = \frac{5}{8} \times \frac{7}{3}
\]

\[
= \frac{35}{24}
\]

Since the GCF of 35 and 24 is 1, the answer cannot be reduced.

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Slide 14/19

Dividing Fractions

Dividing one fraction by another can be done by multiplying the fraction being divided (the dividend) by the reciprocal of the dividing fraction (the divisor).

The reciprocal of a number, \(n\), is simply the value \(\frac{1}{n}\).

When dealing with fractions, this has the result of “flipping” a fraction from \(\frac{a}{b}\) to \(\frac{b}{a}\).

Thus, to evaluate \(\frac{2}{3} \div \frac{4}{5}\), we can instead evaluate the expression \(\frac{2}{3} \times \frac{5}{4}\) instead, reducing first if possible.

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Slide 15/19

Example

Evaluate \(\frac{2}{9} \div \frac{3}{7}\).

Reciprocate \(\frac{3}{7}\) and change the operation to multiplication.

\[
\frac{5}{8} \div \frac{3}{7} = \frac{5}{8} \times \frac{7}{3}
\]

\[
= \frac{35}{24}
\]

Since the GCF of 35 and 24 is 1, the answer cannot be reduced.

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Slide 16/19

Dividing Fractions

Example

Evaluate \(\frac{12}{25} \div \frac{8}{35}\).

Reciprocate \(\frac{8}{35}\) and change the operation to multiplication.

\[
\frac{12}{25} \div \frac{8}{35} = \frac{12}{25} \times \frac{35}{8}
\]

Reduce each fraction, since the GCF of 12 and 8 is 4 and the GCF of 25 and 35 is 5.

\[
\frac{12}{25} \times \frac{35}{8} = \frac{3}{5} \times \frac{7}{2}
\]

\[
= \frac{21}{10}
\]

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Slide 17/19

Dividing Fractions

Example

Evaluate \(\frac{3}{8} \div 9\).

Remember that the reciprocal of 9 is \(\frac{1}{9}\).

\[
\frac{3}{8} \div 9 = \frac{3}{8} \times \frac{1}{9}
\]

The GCF of 3 and 9 is 3, so reduce.

\[
\frac{3}{8} \times \frac{1}{9} = \frac{1}{8} \times \frac{1}{3}
\]

\[
= \frac{1}{24}
\]

J. Garvin — Working with Fractions
Slide 18/19
Questions?