MPM1D: Principles of Mathematics

Working with Fractions

Part 1: Reducing, Multiplying and Dividing

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Order of Operations

Recap

Evaluate $(5-2) \times 4 \cdot 5^2$.

Remember to perform the subtraction and exponentiation before any multiplications.

$$\begin{aligned}(5-2) \times 4 \cdot 5^2 &= 3 \times 4 \cdot 25 \\ &= 12 \cdot 25 \\ &= 300\end{aligned}$$

Greatest Common Factor

Consider the numbers 4 and 10.

Both numbers are even, meaning they are both divisible by 2. In fact, 2 is the largest value that divides evenly into both 4 and 10.

The largest value that divides evenly into two other values is known as the *greatest common factor* (GCF) of those values.

It may be useful to list all factors of each value to determine the GCF. Don't forget 1 and the value itself as factors.

Greatest Common Factor

Example

What is the GCF of 12 and 18?

12 has the factors 1, 2, 3, 4, 6 and 12.

18 has the factors 1, 2, 3, 6, 9 and 18.

Since 6 is the greatest factor shared by both 12 and 18, it is the GCF.

Reducing Fractions

While it is possible to express a fraction as $\frac{12}{20}$, as in "twelve out of twenty people…", the same ratio can be expressed as $\frac{6}{10}$, or "six out of ten people…".

The latter ratio is said to be *reduced*, since the values are smaller.

It is possible to reduce this ratio even further to $\frac{3}{5}$. This ratio is said to be in *simplest form* or *lowest terms*, since the same ratio cannot be expressed any smaller using integers.

Expressing fractions in lowest terms is a mathematical convention, and should be done whenever possible.

To reduce a fraction to lowest terms, both the numerator and the denominator should be divided by their greatest common factor.

Reducing Fractions

Example

Reduce the fraction $\frac{9}{15}$ to lowest terms.

Since the GCF of 9 and 15 is 3, divide both the numerator and denominator by this value.

$$\begin{aligned}\frac{9}{15} &= \frac{9 \div 3}{15 \div 3} \\ &= \frac{3}{5}\end{aligned}$$

Reducing Fractions

Example

Simplify $\frac{21}{40}$.

Factors of 21 are 1, 3, 7, and 21, while factors of 40 are 1, 2, 4, 5, 8, 10, 20 and 40.

Since the GCF of 21 and 40 is 1, it is not possible to reduce the fraction any further.

In this case, the fraction is already in its simplest form, and no further work is necessary.

Improper Fractions

When dealing with mixed fractions, such as $3\frac{1}{2}$, it is usually easier to convert them to improper fractions before multiplying or dividing.

Remember that the numerator of the improper fraction can be made by multiplying the denominator by the whole component, then adding the numerator of the mixed fraction.

The denominator of the improper fraction is the same as the denominator of the mixed fraction.

Thus, $3\frac{1}{2}$ becomes $\frac{7}{2}$, since $2 \times 3 + 1 = 7$.

Improper Fractions

Example

Evaluate $2\frac{3}{4} \times 5\frac{1}{3}$.

Convert each mixed fraction to improper fractions first.

$$\begin{aligned} 2\tfrac{3}{4} \times 5\tfrac{2}{3} &= \frac{11}{4} \times \frac{17}{3} \\ &= \frac{187}{12} \end{aligned}$$

While this can be converted back to a mixed fraction, $15\frac{7}{12}$, it is acceptable (and probably better) to leave it as an improper fraction for the purposes of this course.

Multiplying Fractions

Multiplying two fractions is straightforward enough: multiply the numerators together, and do the same for the denominators.

If it is possible to reduce the resulting fraction to lowest terms, then this should be done.

One problem associated with this direct approach is that either the numerator or denominator of the resulting fraction (or both) may be large, making reduction difficult and time-consuming.

An alternative, then, is to reduce fractions *before* multiplying. This will result in smaller values, potentially making the process easier.

Fractions can be reduced by identifying a GCF of any numerator and any denominator that is greater than 1.

Multiplying Fractions

Example

Evaluate $\frac{2}{15} \times \frac{9}{20}$.

Since the GCF of 2 and 20 is 2, and the GCF of 15 and 9 is 3, reduce these fractions first.

$$\begin{aligned} \frac{2}{15} \times \frac{9}{20} &= \frac{1}{5} \times \frac{3}{10} \\ &= \frac{3}{50} \end{aligned}$$

Compare this to multiplying first, then reducing.

$$\begin{aligned} \frac{2}{15} \times \frac{9}{20} &= \frac{18}{300} \\ &= \frac{3}{50} \end{aligned}$$

While both methods result in the same answer.

Multiplying Fractions

Example

Evaluate $\frac{45}{22} \times \frac{77}{60}$.

Multiplying the fractions directly would give $\frac{3465}{1320}$, which would be difficult to reduce with significant trial-and-error, so reducing first is definitely the better method here.

The GCF of 45 and 60 is 15, while the GCF of 22 and 77 is 11.

$$\begin{aligned} \frac{45}{22} \times \frac{77}{60} &= \frac{3}{2} \times \frac{7}{4} \\ &= \frac{21}{8} \end{aligned}$$

Multiplying Fractions

If it is not easy to identify the GCF from two given values, then reducing by *any* factor will eventually produce the same result after multiple reductions.

For example, if we did not identify 15 as the GCF of 45 and 60, we might start by reducing each value by 5 instead.

$$\frac{45}{22} \times \frac{77}{60} = \frac{9}{2} \times \frac{7}{12}$$

Now reduce both 9 and 12 by 3.

$$\frac{9}{2} \times \frac{7}{12} = \frac{3}{2} \times \frac{7}{4} = \frac{21}{8}$$

The answer is the same as earlier.

Multiplying Fractions

Example

Evaluate $\frac{2}{9} \times 6$.

Rewriting 6 as $\frac{6}{1}$ may make it easier to multiply here. Don't forget to reduce first.

$$\frac{2}{9} \times \frac{6}{1} = \frac{2}{3} \times \frac{2}{1} = \frac{4}{3}$$

Again, it is not necessary to convert $\frac{4}{3}$ as a mixed fraction.

Dividing Fractions

Dividing one fraction by another can be done by multiplying the fraction being divided (the *dividend*) by the *reciprocal* of the dividing fraction (the *divisor*).

The reciprocal of a number, n, is simply the value $\frac{1}{n}$.

When dealing with fractions, this has the result of "flipping" a fraction from $\frac{a}{b}$ to $\frac{b}{a}$.

Thus, to evaluate $\frac{a}{b} \div \frac{c}{d}$, we can instead evaluate the expression $\frac{a}{b} \times \frac{d}{c}$ instead, reducing first if possible.

Dividing Fractions

Example

Evaluate $\frac{5}{8} \div \frac{3}{7}$.

Reciprocate $\frac{3}{7}$ and change the operation to multiplication.

$$\frac{5}{8} \div \frac{3}{7} = \frac{5}{8} \times \frac{7}{3} = \frac{35}{24}$$

Since the GCF of 35 and 24 is 1, the answer cannot be reduced.

Dividing Fractions

Example

Evaluate $\frac{12}{25} \div \frac{8}{35}$.

Reciprocate $\frac{8}{35}$ and change the operation to multiplication.

$$\frac{12}{25} \div \frac{8}{35} = \frac{12}{25} \times \frac{35}{8}$$

Reduce each fraction, since the GCF of 12 and 8 is 4 and the GCF of 25 and 35 is 5.

$$\frac{12}{25} \times \frac{35}{8} = \frac{3}{5} \times \frac{7}{2} = \frac{21}{10}$$

Dividing Fractions

Example

Evaluate $\frac{3}{8} \div 9$.

Remember that the reciprocal of 9 is $\frac{1}{9}$.

$$\frac{3}{8} \div 9 = \frac{3}{8} \times \frac{1}{9}$$

The GCF of 3 and 9 is 3, so reduce.

$$\frac{3}{8} \times \frac{1}{9} = \frac{1}{8} \times \frac{1}{3} = \frac{1}{24}$$

Questions?